REACTIVITY DETERMINATION IN ACCELERATOR DRIVEN REACTORS USING REACTOR NOISE ANALYSIS

by

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Feynman-alpha and Rossi-alpha methods are used in traditional nuclear reactors to determine the subcritical reactivity of a system. The methods are based on the measurement of the mean value, variance and the covariance of detector counts for different measurement times. Such methods attracted renewed attention recently with the advent of the so-called accelerator driven reactors (ADS) proposed some time ago. The ADS systems, intended to be used either in energy generation or transuranium transmutation, will use a subcritical core with a strong spallation source. A spallation source has statistical properties that are different from those traditionally used by radioactive sources. In such reactors the monitoring of the subcritical reactivity is very important, and a statistical method, such as the Feynman-alpha method, is capable of resolving this problem.

Key words: neutron fluctuations, reactor noise, Feynman-alpha method, variance-to-mean ratio, modified variance, subcritical systems, reactivity determination, statistical methods, neutron source, stochastically pulsed source

INTRODUCTION

Various reactions and transport processes of neutrons in a nuclear reactor are random, so the number of neutrons at any time is a random variable [1-3]. Fluctuations in the number of neutrons in a reactor can be divided into two categories: zero reactor noise and power reactor noise. They are predominant at different power levels and the reasons for their occurences and utilization are different. They are also described by different mathematical tools, namely master equations and the Langevin equation, respectively [2, 4].

When a neutron is injected into the system from an extraneous source, it randomly undergoes a number of nuclear events. A particular event might be fission, scattering, capture or detection. The number of neutrons per fission is a random variable. The time between nuclear events is also a random variable. In reactor physics, these neutron fluctuations caused by the above types of sources due to inherent nuclear effects are called zero reactor noise.

In addition to zero noise, large power reactors contain additional noise sources introduced by mechanical perturbations which can arise from temperature and pressure variations, vibrations of control rods and core barrel, formation and transport of steam bubbles in boiling water reactors (BWRs), and so on. The neutron noise induced by such perturbations is referred to as power reactor noise. Power noise carries information about parametric perturbation of the system. Changes in the noise can indicate anomalous changes in the system state or the appearance of new anomalies. That is why the use of power reactor noise for diagnostic purposes is also called neutron noise diagnostics or reactor diagnostics.

Zero noise carries information about some nuclear properties such as reactivity. During 1960ies methods as Feynman-alpha and Rossi-alpha were developed to determine the reactivity of a subcritical system [1]. Such methods attracted renewed interest recently with the appearance of the so-called accelerator driven systems (ADSs). These systems, intended to be used either in energy generation or transuranium transmutation, plan to use

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a subcritical core with a strong spallation source. A spallation source has statistical properties that are different from those of the traditionally used radioactive sources assumed in the derivation of the previous Feynman-alpha and Rossi-alpha formulae. Therefore it was necessary to rederive both formulae. This was actually done recently by several authors [5-9]. However, they all assumed one average delayed neutron group at the most. Recently, the extension of such formulae to a more general case in which six groups of delayed neutron precursors are taken into account, and full joint statistics of the prompt and all delayed groups were included in papers [10-12]. These papers contain the most complete list of formulae with an explicit use of six delayed neutron groups as well as full prompt-prompt, prompt-delayed and delayed-delayed correlations and the multiplicity of the source.

In a spallation source, which will be used in an accelerator driven system, all neutrons arising from the spallation reactions of one primary projectile, usually a proton, are correlated, so the source statistics is not just Poisson, but composed of a Poisson distribution of the emission event and a Gaussian distribution of the source generating neutrons. Although spallation neutrons arisen by one projectile are born within a finite time span and not simultaneously, this time span is very short (a few nanoseconds) in comparison with the generation time of fission neutrons in the fission chain, and it is even shorter than the lifetime of the prompt neutron chain. The arrival time of the projectile (*e. g.* a proton) is assumed to follow a compound Poisson statistics. In other words, it means that there will be a correlations between the generated neutrons, in contrast to the traditional case, not only in the fission chain but also in the external source.

TRADITIONAL FEYNMAN-ALPHA AND ROSSI-ALPHA FORMULAE

Zero noise carries information about nuclear properties of the system such as reactivity. Therefore two fluctuation based methods, the Feynman-alpha or the variance-to-mean, and the Rossi-alpha or the covariance method have been extensively used to measure reactor subcritical reactivity [1-4]. They are both based on the measurement of the second moment of the statistics of the detector counts. In the Feynman method, one determines the relative variance

$$\frac{\sigma_Z^2(t)}{\langle Z(t)\rangle} \qquad (1)$$

as a function of the measurement time t. Here, $Z(t)$ is the detector count in $(0, t)$, a random variable, $\langle Z(t)\rangle$ is its expected value and $\sigma_Z^2(t)$ its variance. By means of the theory of linear Markov processes, the following expression was derived earlier for the variance-to-mean (Feynman-alpha formula) [13]:

$$\frac{\sigma_Z^2(t)}{\langle Z(t)\rangle} = 1 + \frac{\widetilde{\mu}_{ZZ}(t)}{\widetilde{Z}(t)} =$$

$$= 1 + \varepsilon \sum_{i=0}^{6} A_i \left[1 - \frac{1 - e^{-\alpha_i t}}{\alpha_i t}\right] = 1 + Y(t) \qquad (2)$$

In the above equation, ε is the detector efficiency. The most important part of the sum is the prompt part, *i. e.*, $i = 0$, for which one has

$$\alpha_0 \approx \frac{\beta - \rho}{\Lambda}, \quad \rho = \frac{k-1}{k}, \quad \alpha_i \ll \alpha_0; \ i \geq 1 \qquad (3)$$

and

$$A_0 = \frac{\langle \nu(\nu-1)\rangle}{\bar{\nu}^2} \frac{1}{(\beta-\rho)^2} \equiv \frac{D_\nu}{(\beta-\rho)^2} \qquad (4)$$

where β is the delayed neutron fraction, ρ the subcritical reactivity, and Λ the prompt neutron generation time. The quantities $\bar{\nu}$, $\langle \nu(\nu-1)\rangle$, and $D_\nu = \langle \nu(\nu-1)\rangle / \bar{\nu}^2$ are related to the various moments of the number distribution of the fission neutrons [10]. For the sake of simplicity, the bar from $\bar{\nu}$ will be omitted in the following text. It is necessary to note that eq. (2) is derived with the assumption of a Poisson source, *i. e.*, the probability of emitting *one* neutron in dt is $S\,dt$.

Physical interpretation of eq. (2) is as follows. If all neutrons were statistically independent, as *e. g.*, those emitted by a radioactive source, the statistics would be Poisson and relative variance equal to unity. However, in a multiplying medium, each neutron will induce a chain, leading to the generation of a total on $1/(1 - k)$ neutrons in an infinitive system. All neutrons in such a chain are correlated due to the fact that they have a common origin. Due to positive correlations, the variance will be higher than Poisson. Since each individual chain will die out in a subcritical reactor, the die out being determined by the time constants α_i, relative variance will saturate. It is in this part of the variance-to-mean which exceeds unity where the useful information on the system is found.

The Rossi-alpha method, based on the measurement of the covariance function of the detector counts in infinitesimal time intervals dt around times t and $t + \tau$, is defined as

$$P(\tau)d\tau = \varepsilon d\tau \sum_{i=0}^{6} C_i e^{-\alpha_i \tau} \qquad (5)$$

The time constants α_i are the same as in eqs. (2) and (3), and the constants C_i similar to A_i in eq. (4).

The interpretation of eq. (5) is also similar to that of the Feynman-alpha. If all neutrons were independent, the covariance would be zero, but it dies out exponentially with the same time constants as the ones that appear in the Feynman-alpha formula. Again, the useful information on the system such as its reactivity is contained in the exponents α_i, in particular in α_0.

GENERALIZATION OF THE FEYNMAN-ALPHA AND ROSSI-ALPHA FORMULAE

The Feynman-alpha and Rossi-alpha formulae had to be generalized in order to be applied to the spallation driven subcritical system because of the different statistical properties of the external source. This has been done recently by several authors [5-9]. However, in all these papers a single average delayed neutron group was assumed (in some papers the delayed neutrons were not even explicitly taken into account). Paper [10] consists of the generalisation of the method to the case when six different delayed neutron group are distinguished.

The full joint statistics is explicitly expressed in the following Feynman-alpha (6) and Rossi-alpha (12) formulae

$$Y(t)=\frac{\tilde{\mu}_{ZZ}(t)}{\tilde{Z}(t)}=2\varepsilon\lambda_f^2\left[\langle\nu_p(\nu_p-1)\rangle+\right.$$

$$\left.+\langle q(q-1)\rangle\frac{\nu}{\bar{q}}(-\rho)\right]\sum_{i=0}^{6}\omega_i f_i(t)-$$

$$-2\varepsilon\lambda_f^2\sum_{j=1}^{6}\lambda_j^2(2\langle\nu_p\nu_d\rangle+$$

$$+\langle\nu_{d_j}(\nu_{d_{jj}}-1)\rangle)\sum_{i=0}^{6}\frac{\omega_i f_i(t)}{s_i^2-\lambda_j^2}-$$

$$-4\varepsilon\lambda_f^2\sum_{\substack{i,j=1\\ i<j}}\frac{\lambda_i\lambda_j}{\lambda_i+\lambda_j}\langle\nu_{d_i}\nu_{d_j}\rangle\cdot$$

$$\cdot\left[\lambda_i\sum_{k=0}^{6}\frac{\omega_k f_k(t)}{s_k^2-\lambda_i^2}+\lambda_j\sum_{k=0}^{6}\frac{\omega_k f_k(t)}{s_k^2-\lambda_j^2}\right] \quad (6)$$

Here, $\tilde{Z}(t)$ is the expected value of the detector counts in a stationary statistical system between time 0 and t, $\tilde{\sigma}_{ZZ}^2$ is its variance, and $\tilde{\mu}_{ZZ}(t)=\tilde{\sigma}_{ZZ}^2(t)-\tilde{Z}(t)$ is the modified variance.

Here, besides the standard notations, the following notations were used:

$$\omega_i\equiv\frac{z_i}{s_i}\sum_{j=0}^{6}\frac{z_j}{s_i+s_j},\quad i=0,1,\ldots,6 \quad (7)$$

$$f_i(t)\equiv\left(1+\frac{1-e^{s_i t}}{s_i t}\right)=\left(1-\frac{1-e^{-\alpha_i t}}{\alpha_i t}\right),\ (\alpha_i=-s_i) \quad (8)$$

and

$$z_k=\frac{\prod_{j=1}^{6}(s_k+\lambda_j)}{\prod_{j\neq k}(s_k-s_j)}=$$

$$=\frac{1}{\left(1+\frac{1}{\Lambda}\sum_{j=1}^{6}\frac{\beta_j\lambda_j}{(s_k+\lambda_j)^2}\right)} \quad (9)$$

The s_k, $k = 0, 1, \ldots, 6$, are the solutions of the characteristic equation (the inhour equation):

$$s+\alpha-\frac{1}{\Lambda}\sum_{j=1}^{6}\frac{\beta_j\lambda_j}{s+\lambda_j}=0 \quad (10)$$

where

$$\alpha\equiv-\frac{\rho-\beta}{\Lambda}>0 \quad (11)$$

The Rossi-alpha formula is given in the following form:

$$P_{rossi}(t)dt=\varepsilon\lambda_f^2 dt\cdot$$

$$\left\{\left[\langle\nu_p(\nu_p-1)\rangle+\langle q(q-1)\rangle\frac{\nu}{\bar{q}}(-\rho)\right]\sum_{i=0}^{6}\omega_i g_i(t)-\right.$$

$$-\sum\lambda_j^2(2\langle\nu_p\nu_{d_j}\rangle+\langle\nu_{d_j}(\nu_{d_j}-1)\rangle)\sum_{i=0}^{6}\frac{\omega_i g_i(t)}{s_i^2-\lambda_j^2}-$$

$$\left.-2\sum_{\substack{i,j=1\\ i<j}}\frac{\lambda_i\lambda_j}{\lambda_i+\lambda_j}\langle\nu_{d_i}\nu_{d_j}\rangle\left[\lambda_i\sum_{k=0}^{6}\frac{\omega_k g_k(t)}{s_k^2-\lambda_i^2}+\lambda_j\sum_{k=0}^{6}\frac{\omega_k g_k(t)}{s_k^2-\lambda_j^2}\right]\right\}$$

(12)

In the above, all notations are the same as with the Feynman-alpha formula. The difference is that the functions $f_i(t)$ are replaced by the functions $g_i(t)$ that are defined as

$$g_i(t)\equiv-s_i e^{s_i(t)}=\alpha_i e^{-\alpha_i t},\quad i=0,1,\ldots,6$$

REACTIVITY DETERMINATION IN A MULTIPLYING SYSTEM WITH A STOCHASTICALLY PULSED NEUTRON SOURCE

The Feynman-alpha (variance-to-mean) method is used in traditional nuclear reactors to determine the subcritical reactivity of a system. Steady-state neutron flux is maintained in the reactor core during the measurement by an external neutron source, usually a radioactive source. It is a method that uses neutron fluctuations and does not perturb the reactor. Cross sections, detector efficiency and source strength need not be known. From a plot of the variance-to-mean ratio as a function of measurement time, the reactivity is determined by fitting the measured curve to the analytical solution.

For a continuous source, without delayed neutrons, the variance-to-mean has a simple expression

$$\frac{\sigma_Z^2(t)}{\langle Z(t)\rangle}=1+\varepsilon A\left[1-\frac{1-e^{-\alpha t}}{\alpha t}\right]\equiv 1+Y(t) \quad (13)$$

where

ε – is detector efficiency, $\varepsilon = \lambda_d/\lambda_f$,

λ_d – is the probability of detection per neutron and unit time, $\lambda_d \equiv \mathrm{v}\Sigma_d$,

v – is the constant neutron velocity,

Σ_d – is the macroscopic cross section of detection,

λ_f – is the probability of fission per time unit per neutron, $\lambda_f \equiv \mathrm{v}\Sigma_f$, and

Σ_f – is the macroscopic cross section of fission.

The shape of the $Y(t)$ graph depends on α and

$$\alpha=-\rho/\Lambda \quad (14)$$

where

ρ – is the reactivity, and

Λ – is the prompt generation time.

Detector efficiency ε and the constant A appear only as factors and do not affect the very shape of the curve.

The concept of the so-called accelerator driven subcritical reactors (ADS), proposed some time ago, has received substantial interest recently. The essence is a subcritical system driven by a very strong external neutron source, based on an accelerator. Such a reactor can transmute radioactive waste, and also utilize fertile/fissionable material, mostly ^{232}Th, as fuel. In such reactors the monitoring of the subcritical reactivity is very important, and a statistical method, such as the Feynman-alpha method, is capable to resolve this problem. However, it was necessary to develop a new theory taking into account the statistical properties of the new source, different from a traditional radioactive source.

In some recently started European Union projects, it is planned that, among other methods, the Feynman-alpha method will be used for reactivity monitoring. Therefore, in case of a pulsed accelerator, the variance and mean values of detected neutrons have to be re-derived.

The Feynman-alpha method uses different measurement times, so the measurement data, taken during the long period continuously, need to be devided into blocks. There are two ways of dividing data into blocks of different lenght. They are called deterministic and stochastic pulsing. Deterministic pulsing means that data blocks are taken so that the beginning of a block always coincides with the start of a pulse from the neutron generator. The second way of dividing the data into blocks does not use any synchronisation between the start of the measurement time and the pulsing. There is a randomness in the measurement start. For calculation purposes the randomness can be represented in the source, so that there is a stochastic variable for the first switching on of the source.

With a pulsed source the calculations for the variance-to-mean have to be re-done. Therefore, the pulsation of the neutron generator is modelling with

$$S(t)=S\sum_{n=-\infty}^{\infty}[H(t-nT_0)-H(t-W-nT_0)] \quad (15)$$

where T_0 is the pulse period, W the pulse width, S the pulse amplitude and H denotes the unit step function. The probability of emitting one source neutron within $(t, t+dt)$ is $S(t)\,dt$.

In this paper, we will concentrate on stochastic pulsing, that is, on the case when there is no synchronisation between the measurement start and the incoming pulse, *i. e.*, the start of the measurement time is random. This is equivalent to fixing the time axis at the measurement start and to describe a source with a random function [14]

$$S(t,\xi)=S\sum_{n=-\infty}^{\infty}[H(t-nT_0-\xi)-$$
$$-H(t-W-nT_0-\xi)] \quad (16)$$

where ξ is a random number, distributed uniformly in $(0, T_0)$. Thus the source term will correspond to a time dependent Poisson distribution whose parameter $S(t)$ will contain a random element. This is called stochastic pulsing or random pulsing.

Derivation of the modified variance of detected neutrons in a multiplying system with a stochastically pulsed Poisson source

To calculate the modified variance of emitted neutrons from a stochastically pulsed Poisson source, we begin from the following balance equation:

$$P(n,t|t_0) = [1 - S(t_0,\xi)dt_0]P(N,t|t_0 + dt_0) +$$

$$+S(t_0,\xi)dt_0 P(N-1,t|t_0 + dt_0) \quad (17)$$

The left hand side of the upper relation means the probability of having N neutrons in the system at time t, given that there were no neutrons at the time t_0. This probability is equal to the sum of the probabilities for the two mutually exclusive events at the right hand side:

(1) get no neutron during dt_0 and get N neutrons from $t_0 + dt_0$ to t, and

(2) get one neutron during dt_0 and get $N - 1$ neutrons from $t_0 + dt_0$ to t.

With the initial condition

$$P(N,t_0|t_0) = \delta_{N,0} \quad (18)$$

from the balance eq. (17) we receive the following equation

$$-\frac{d}{dt_0}P(N,t|t_0) =$$

$$= -S(t_0,\xi)P(N,t|t_0) + S(t_0,\xi)P(N-1,t|t_0) \quad (19)$$

For square pulses, starting from the upper equation, the mean value of emitted neutrons is equal to

$$\langle N(t)\rangle = \frac{SW}{T_0}t \quad (20)$$

Compared with the mean value for a continuous source

$$\langle N(t)\rangle = St \quad (21)$$

the mean value for a stochastic source is scaled down with the factor W/T_0.

The modified variance defined as

$$\mu_N = \langle N(N-1)\rangle - \langle N\rangle^2 \quad (22)$$

is given by the following eq. [14]

$$\mu_N = \frac{1}{T_0}\int_0^t\int_0^t\int_0^{T_0} S(t',\xi)S(t'',\xi)dt'dt''d\xi -$$

$$-\left(\frac{SW}{T_0}\right)^2 t^2 \quad (23)$$

For the Feynman-alpha formula in a multiplying system one will need the asymptotic first and second moments of the source induced detector count $\widetilde{Z}(t)$.

However, it is easy to show that, with the source given by eq. (16), for any t

$$\frac{1}{T_0}\int_0^{T_0} S(t,\xi)d\xi = \frac{SW}{T_0} \quad (24)$$

and that

$$\langle\widetilde{Z}(t)\rangle = \frac{SW}{T_0}\int_0^t Z(t-t',T)dt' \quad (25)$$

The calculation of the second moment $M_{\widetilde{Z}}(t,T)$ is more complicated and will not be proportional to that of the continuous source.

The modified variance is defined as

$$\mu_{\widetilde{Z}}(t,T) \equiv M_{\widetilde{Z}}(t,T) - \widetilde{Z}^2 \quad (26)$$

and will be

$$\mu_{\widetilde{Z}}(t,T) = \int_0^t\int_0^t K(t',t')Z(t',T)Z(t'',T)dt'dt'' +$$

$$+\frac{SW}{T_0}\int_0^t M_Z(t',T)dt' - \left(\frac{SW}{T_0}\right)^2\left[\int_0^t Z(t',T)dt'\right]^2 \quad (27)$$

where

$$K(t',t'') \equiv \frac{1}{T_0}\int_0^{T_0} S(t',\xi)S(t'',\xi)d\xi \quad (28)$$

Calculation of the variance-to-mean of the stochastically pulsed source

According to the standard definition of the variance, $Var(N)$, variance-to-mean ratio, $Var(N)/\langle N\rangle$, for neutrons emitted from the stochastically pulsed source is equal to

$$\frac{Var(N)}{\langle N\rangle} = \frac{\langle N(N-1)\rangle + \langle N\rangle - \langle N\rangle^2}{\langle N\rangle} =$$

$$= \frac{\langle N(N-1)\rangle}{\langle N\rangle} + 1 - \langle N\rangle \quad (29)$$

Since it is only the part of the variance-to-mean that exceeds that of a Poisson source that is interesting, usually the modified variance is used instead

$$\frac{\mu_N}{\langle N\rangle} = \frac{\langle N(N-1)\rangle - \langle N\rangle^2}{\langle N\rangle} \quad (30)$$

The numerator in the upper relation is the difference between $\langle N(N-1)\rangle$ and the squared first moment (20).

The second moment, $\langle N(N-1)\rangle$, has been calculated and the following formula for the modified variance-to-mean is received

$$\frac{\mu_N}{\langle N\rangle}=\frac{\langle N(N-1)\rangle-\langle N\rangle^2}{\langle N\rangle^2}=$$

$$=\frac{\left(\frac{SW}{T_0}\right)^2 t^2+2\sum_{n=1}^{\infty}\left(\frac{S^2T_0^2}{n^4\pi^4}\right)\left(\sin\frac{n\pi t}{T_0}\right)^2\left(\sin\frac{n\pi W}{T_0}\right)^2}{\frac{SW}{T_0}t}-$$

$$-\frac{\left(\frac{SW}{T_0}t\right)^2}{\frac{SW}{T_0}t}=2\frac{ST_0^3}{\pi^4 W}\sum_{n=1}^{\infty}\frac{1}{n^4 t}\left(\sin\frac{n\pi t}{T_0}\right)^2\left(\sin\frac{n\pi W}{T_0}\right)^2 \qquad (31)$$

It is interesting to note that the sum in eq. (31) decreases as $1/n^4$. Therefore a good approximation for the modified variance-to-mean can be achieved by taking only the first three terms in the summation.

THE MODIFIED VARIANCE IN A MULTIPLYING SYSTEM WITH A STOCHASTICALLY PULSED NEUTRON SOURCE

With the aid of the complex function theory method and Laplace transform of a convolution, the following solution for the first three terms of the modified variance for the measurement time T was received [14]:

$$\mu_{\tilde{Z}}(T)=\frac{\lambda_d^2 S^2 W^2}{\alpha^2 T_0^2}T^2+$$

$$+2\sum_{n=1}^{\infty}\frac{\lambda_d^2 S^2 T_0^4}{n^4\pi^4(4n^2\pi^2+\alpha^2T_0^2)}\left(\sin\frac{n\pi T}{T_0}\right)^2\cdot$$

$$\cdot\left(\sin\frac{n\pi W}{T_0}\right)^2+\frac{SW}{T_0}\lambda_f\langle\nu(\nu-1)\rangle\frac{\lambda_d^2}{\alpha^4}\cdot$$

$$\cdot(\mathrm{e}^{-\alpha T}+\alpha T-1)-\frac{S^2W^2}{T_0^2}\frac{\lambda_d^2}{\alpha^2}T^2 \qquad (32)$$

The first term of the upper equation and the last one cancel each other and

$$\mu_{\tilde{Z}}(T)=2\sum_{n=1}^{\infty}\frac{\lambda_d^2 S^2 T_0^4}{n^4\pi^4(4n^2\pi^2+\alpha^2T_0^2)}\cdot$$

$$\cdot\left(\sin\frac{n\pi T}{T_0}\right)^2\left(\sin\frac{n\pi W}{T_0}\right)^2+$$

$$+\frac{SW}{T_0}\lambda_f\langle\nu(\nu-1)\rangle\frac{\lambda_d^2}{\alpha^4}(\mathrm{e}^{-\alpha T}+\alpha T-1) \qquad (33)$$

Dividing the upper relation with the mean value, we get the modified variance-to-mean in the following form:

$$\frac{\mu_{\tilde{Z}}(T)}{\langle\tilde{Z}\rangle}=2\sum_{n=1}^{4}\frac{\lambda_d^2 S^2 T_0^4}{n^4\pi^4(4n^2\pi^2+\alpha^2T_0^2)}\cdot$$

$$\cdot\left(\frac{SW}{T_0}\frac{\lambda_d}{\alpha}T\right)^{-1}\left(\sin\frac{n\pi T}{T_0}\right)^2\left(\sin\frac{n\pi W}{T_0}\right)^2+$$

$$+\frac{SW}{T_0}\lambda_f\langle\nu(\nu-1)\rangle\frac{\lambda_d^2}{\alpha^4}(\mathrm{e}^{-\alpha T}+$$

$$+\alpha T-1)/\left(\frac{SW}{T_0}\frac{\lambda_d}{\alpha}T\right) \qquad (34)$$

which gives

$$\frac{\mu_{\tilde{Z}}(T)}{\langle\tilde{Z}\rangle}=2\frac{\lambda_d S T_0^5\alpha}{t\pi^4 W}\sum_{n=1}^{\infty}\frac{1}{(4n^6\pi^2+n^4\alpha^2T_0^2)}\cdot$$

$$\cdot\left(\sin\frac{n\pi T}{T_0}\right)^2\left(\sin\frac{n\pi W}{T_0}\right)^2+$$

$$+\frac{\lambda_d\lambda_f}{\alpha^2}\langle\nu(\nu-1)\rangle\left(1-\frac{1-e^{-\alpha T}}{\alpha T}\right) \qquad (35)$$

The upper relation represents the basic relation for the reactivity monitoring in a multiplying system with a stochastically pulsed neutron source.

RESULTS

The last term in the relation (35) has the same dependence on T as the variance-to-mean for a continuous source (2). In fact it is identical to the Feynman-Y value for a continuous source.

The modified variance-to-mean formula (35) has a strong dependence on the pulsation period of the neutron generator, T_0, in the first term. However, the second term has no dependence on T_0 or the pulse width, W. That means that the second term is totally independent of the pulsation properties.

The function $Y(T) = \mu_{\widetilde{Z}}(T)/\langle\widetilde{Z}\rangle$ for the pulsing source is shown in Fig. 1 for one particular case.

If we let the period time and the pulse width increase by factor two, we will get the graph shown in Fig. 2.

The pulsation period and the pulse width are thus important parameters for the experiment. If the oscillations in the variance-to mean are too big the method may be less useful for the determination of reactivity.

If we keep the same period time and decrease the pulse width then the amplitude on the oscillations will decrease. This is due to the last sine factor in eq. (35). An example is shown in Fig. 3.

The lower envelope of the pulsed curve is given by the traditional smooth Feynman-alpha expression. To determine reactivity we can fit a curve to the local minimum and read out the reactivity parameter. It may also be possible to fit a curve to the complete experimental curve.

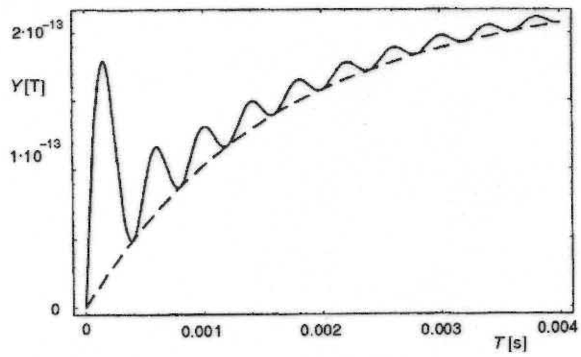

Figure 1. Time dependence of the modified variance-to-mean for a continuous (dashed) and a stochastic pulsing source (T_0= 0.0004 s, W = 0.0002 s)

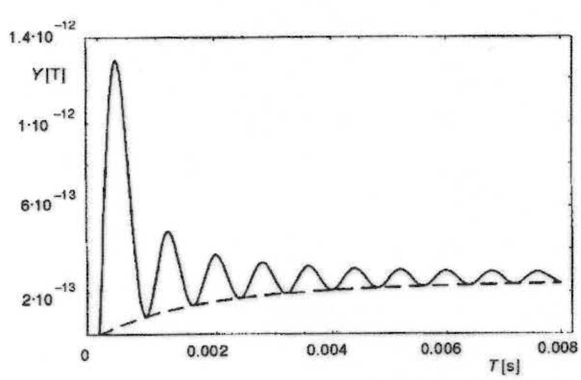

Figure 2. Time dependence of the modified variance-to-mean for a continuous (dashed) and a stochastic pulsing source (T_0 = 0.0008 s, W = 0.0004 s)

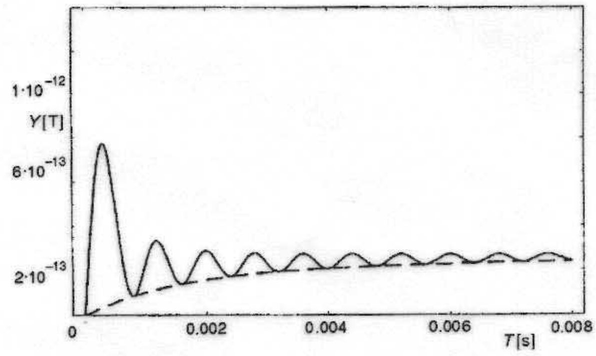

Figure 3. Time dependence of the modified variance-to-mean for a continuous (dashed) and a stochastic pulsing source (T_0 = 0.0008 s, W = 0.0001 s)

CONCLUSIONS

The Feynman-alpha method is successfully used in traditional nuclear reactors to determine the subcritical reactivity of a system. Measurement is performed while a steady-state neutron flux is maintained in the reactor by an external neutron source, as a rule a radioactive source.

A new situation arises in the planned accelerator driven systems (ADSs). An ADS system will be run in a subcritical mode. Steady flux will be maintained by an accelerator-based source with statistical properties that are different from those of a steady radioactive source.

In this paper, an analytical Feynman-alpha formula for the case of stochastic pulsing is presented and analysed. The obtained results show that stochastic pulsing gives a variance-to-mean curve that is smoothly regular with a simple periodic oscillation. It consists of a Feynman curve corresponding to a stationary source, plus an infinite sum of periodic sine functions squared. Thus the traditional smooth Feynman-alpha expression is given as the lower envelope of the pulsed curve. This result is suitable for the determination of the subcritical reactivity in the future ADS systems by fitting a curve to the local minima of the variance-to-mean experimental curve, or even to the complete curve.

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Љиљана КОСТИЋ

ОДРЕЂИВАЊЕ РЕАКТИВНОСТИ У АКЦЕЛЕРАТОРОМ ПОБУЂИВАНИМ РЕАКТОРИМА КОРИШЋЕЊЕМ АНАЛИЗЕ РЕАКТОРСКОГ ШУМА

Фејнман-алфа и Роси-алфа методе коришћене су у традиционалним нуклеарним реакторима за одређивање поткритичне реактивности система. Методе се заснивају на мерењу средње вредности варијансе и коваријансе откуцаја детектора за различита времена мерења. Недавно, са појавом идеје о тзв. акцелератором побуђиваним реакторима, обновљен је интерес за овим методама. Ови реактори, намењени да се користе за производњу енергије или трансмутацију радиоактивног отпада употребљаваће јаке спалационе изворе чија се статистичка својства разликују од својстава традиционално коришћених радиоактивних извора. У таквим реакторима праћење поткритичне реактивности је веома важно и статистичка метода као Фејнман-алфа је способна да реши тај проблем.