ANGULAR DISTRIBUTION AND INERTIA PARAMETERS IN ALPHA-INDUCED FISSION OF ^{232}Th, ^{233}U, AND ^{238}U

by

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The analysis of the selected fission fragment angular distribution from alpha induced fission is made by using an exact theoretical expression. Theoretical anisotropies obtained with transition state modes are compared with their corresponding experimental values. The agreement between the calculated and experimental values is very good. The values of the statistical parameter K_0^2 are used for calculation of the inertia parameters. The results indicate an increase in the moment of inertia.

Key words: fragment angular distribution in (α, f) reaction, statistical scission model

INTRODUCTION

There are considerable evidences that the Statistical Transition Model (TSM) provides a good representation of experimental fission fragment angular distributions at low spin values and moderate excitation energies. The fundamental assumption of this model is the spin projection K, on the nuclear symmetry axis, which remains unchanged during the fission process [1]. In heavy reaction systems, where the angular momentum and excitation energy are large, fission fragment angular distributions were analyzed by the Statistical Scission Model (SSM). Rossner *et al.*, [1] and Bond [2] have published some versions of this model. Although the formal equations in the two models have the same structure, variances in the distributions of angular momentum projections on the fission direction are established at very different stages of the fission process in the two models. Various authors studied the properties of complex transition state [3, 4]. Nuclear moments of inertia were extracted from the measured fission fragment anisotropies in the paper of Huizenga *et al.* [5, 6].

In the present work, we have developed a special computer code to deduce the statistical parameter K_0^2 ($K_0^2 = J_{eff}T/\hbar^2$) from experimental angular anisotropies using the exact theoretical expressions.

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FORMALISM OF TRANSITION STATE MODEL AT MODERATE EXCITATION ENERGY

The excited levels in the transition nucleus are described by statistical theory. The K-distribution of these levels is predicted to be Gaussian by Halpen *et al.* [7]:

$$F(K) \propto \exp\left(-\frac{K^2}{2K_0^2}\right) \tag{1}$$

with a variance of

$$K_0^2 = \frac{J_{eff}T}{\hbar^2} \tag{2}$$

The effective moment of inertia is $J_{eff} = J_\parallel J_\perp / (J_\parallel + J_\perp)$ where $J_\perp$ and $J_\parallel$ are nuclear moments of inertia at the axis perpendicular and parallel to the symmetry axis respectively, and T is the temperature of the nucleus in the transition state.

Assuming that the fragments separate along the symmetry axis, and that K is a good quantum number during the fission process, the angular distribution of fragments from a state with the quantum numbers K and M (projection of the total spin I along the space fixed axis) is given by [8]

$$W^{I}_{M,K}(\theta)=\frac{2I+1}{4\pi}\left|d^{I}_{M,K}(\theta)\right|^{2} \qquad (3)$$

The normalized $d^{I}_{M,K}(\theta)$ functions are defined by [9]:

$$d^{I}_{M,K}(\theta)=[(I+M)!(I-M)!(I+K)!(I-K)!]^{1/2}\cdot$$

$$\cdot\sum_{x}\frac{(-1)^{x}(\sin\theta/2)^{K-M+2x}(\cos\theta/2)^{2I-K+M-2x}}{(I-K-x)!(I+M-x)!(x+k-M)!x!} \qquad (4)$$

where the sum runs over $x = 0, 1, 2, \ldots$ and contains all terms in which no negative value appears in the denominator of the sum for any of the quantities in parentheses.

If the target and projectile spins are zero and no particle emission from the initial compound nucleus occurs before fission (*i. e.* $M = 0$), then the overall angular distribution for fixed energy E is given by Griffin [10]:

$$W(\theta)\propto\sum_{I=0}^{\infty}(2I+1)T_{\mathrm{l}}\cdot$$

$$\cdot\sum_{K=-I}^{I}\left[(2I+1)\left|d^{I}_{M=0,K}(\theta)\right|^{2}\cdot e^{g(K,K_0)}\right] \qquad (5)$$

where the transmission coefficients are written as T_{l}, ($M = 0$ gives $I = \mathrm{l}$), and

$$g(K,K_0)=\frac{\dfrac{-K^2}{2K_0^2}}{\sum\limits_{K=-I}^{I}e^{\frac{-K^2}{2K_0^2}}}$$

Equation (5) is an exact theoretical expression for computation of the fission fragment angular distribution when both the target and projectile spins are zero. If the target and projectile spins are included, an exact expression for the angular distribution of fission fragments is [9, 10]:

$$W(\theta)\propto\sum_{I=0}^{\infty}\sum_{M=-J_{\min}}^{J_{\max}}\cdot$$

$$\cdot\sum_{l=0}^{\infty}\sum_{j=|l-s|}^{l+s}\sum_{\mu=-l}^{l}\frac{(2l+1)T_l\left|C^{j,l,I}_{M,0,M}\right|^2\left|C^{I_0,s,l}_{\mu,M-\mu,M}\right|^2}{\sum\limits_{l=0}^{\infty}(2I+1)T_l}\cdot$$

$$\cdot\sum_{K=-I}^{I}\left[(2I+1)\left|d^{I}_{M,K}(\theta)\right|^{2}\cdot e^{g(K,K_0)}\right] \qquad (6)$$

The quantities I_0, s, and J are the target spin, projectile spin, and channel spin, respectively ($J = I_0 \oplus s$). The total angular momentum I is given by the sum of the channel spin and orbital angular momentum $I = J \oplus l$. The projection of I_0 on the space-fixed axis is given by μ, and the projection of J (and I) on this axis is M.

FORMALISM OF THE STATISTICAL SCISSION MODEL

According to the SSM, the relative cross-section $w(\theta)$, for fission fragments to be emitted in the direction $\hat{n}$ forming angle θ with the beam axis when the target projectile spins are zero, is given by Huizenga *et al.* [11]:

$$W(\theta)\propto\sum_{I=I_{\min}}^{I_{\max}}(2I+1)T_I\cdot$$

$$\cdot\sum_{K=-I}^{I}\left[(2I+1)\left|d^{I}_{M=0,K}(\theta)\right|^{2}\cdot e^{g(s,S_0)}\right] \qquad (7)$$

Again, the distribution of spin projection M (the projection of the total angular momentum I along $\hat{n}$) is taken to be a Gaussian with variance S_0^2, where S_0^2 for spherical fission fragment is given by either of the following equations [12]:

$$S_0^2=2I_{sph}\frac{T}{\hbar^2}\frac{2I_{sph}+\mu R_c^2}{\mu R_c^2} \qquad (8)$$

or

$$S_0^2=2\sigma^2\frac{2\sigma^2+\mu R_c^2\dfrac{T}{\hbar^2}}{\mu R_c^2\dfrac{T}{\hbar^2}} \qquad (9)$$

with $\sigma^2 = I_{sph}T/\hbar^2 = 0.4MR^2T/\hbar^2$. The quantities I_{sph}, T, M, and R are the moment of inertia, nuclear temperature, mass, and radius of one of the symmetric fission fragments, while R_c is the distance between the centers of fragments at scission configuration and is equal to $1.225(A_1^{1/3}+A_2^{1/3})(c/a)^{2/3}$ (A_1 and A_2 are mass numbers of fission fragments).

For a scission configuration of the two unattached deformed fragments, the variance S_0^2 is given by either of the two equations [12]:

$$S_0^2=\frac{2I_{\parallel}\dfrac{T}{\hbar^2}}{\dfrac{2I_{\perp}+\mu R_c^2}{2I_{\perp}+\mu R_c^2-2I_{\parallel}}} \qquad (10)$$

or

$$S_0^2 = 2\sigma_\parallel^2 \frac{2\sigma_\perp^2 + \mu R_c^2 \frac{T}{\hbar^2}}{2\sigma_\perp^2 + \mu R_c^2 \frac{T}{\hbar^2} - 2\sigma_\parallel^2} \quad (11)$$

where $I_\parallel, I_\perp$, $\sigma_\parallel^2$ and $\sigma_\perp^2$ are moments of inertia and spin cutoff parameters of a single fission fragment rotating about an axis parallel and perpendicular to the symmetry axis, respectively.

The primary fission fragments are assumed to have nonspheroidal shapes with the principal one-half axes of magnitude in terms of their ratio c/a, namely $a = 1.225(A_c/2)^{1/3}(c/a)^{-1/3}$ and $c = 1.225\ (A_c/2)^{1/3}(c/a)^{2/3}$, where A_c is the mass number of the composite system. The total intrinsic excitation energy of two fission fragments at scission is given by [13]:

$$E^* = E_{c.m} + Q - E_{Coul} - E_{def} - E_{rot} \quad (12)$$

where Q represents the difference in energy between the entrance channel nuclei and the ground state of the two fission fragments. Here, $E_{Coul} + E_{def}$ is the sum of the Coulomb and deformation energies stored in potential energy at the instant scission. The Coulomb energy is estimated by using the expression [13]:

$$E_{Coul}[\text{MeV}] = 1.44 \frac{Z^2}{2c} \quad (13)$$

where Z is one-half of the charge of the composite system. The rotational energy E_{rot} of the system at the scission configuration for spin I and projection m on the scission axis is [14]:

$$E_{rot} = \frac{\left[\left(I + \frac{1}{2}\right)^2 - m^2\right]\hbar^2}{2\mu c^2 + 4I_\perp} \quad (14)$$

where μ is the reduced mass of the fission fragments. The temperature of each fission fragment was assumed to be given by:

$$T = \left[\frac{E^*/2}{LDP}\right]^{1/2} \quad (15)$$

where LDP is the liquid drop parameter, A is the mass number of one fragment, and the total excitation energy $E^*$is divided equally between the two symmetric fission fragments.

The TSM and SSM are formulated according to this assumption, by the rules of which the fission will occur after combination compound nucleus. Therefore, these models enable the explanation of the experimental angular distribution of fission fragments for heavy or light nuclei and also for heavy ion reactions if they provide this assumption.

RESULTS AND DISCUSSIONS

In this paper, the angular distributions of fission fragments in alpha induced fission of ^{232}Th, ^{233}U, and ^{238}U are studied by TSM and SSM. Both of these models depend on the projectile energy and spins of projectile and target, if the angular distributions of fission fragments are to be explained. To consider the spin effects and the effects of different energies in these models, we have studied several different targets such as ^{232}Th(0^+), ^{233}U($5/2^+$) and ^{238}U(0^+) under alpha induced fission in several alpha energies.

Variances K_0^2 (or S_0^2) for alpha induced fission reactions of ^{232}Th and ^{238}U considering vanishing spins of alpha, ^{232}Th, and ^{238}U, have been obtained by fitting experimental angular distribu-

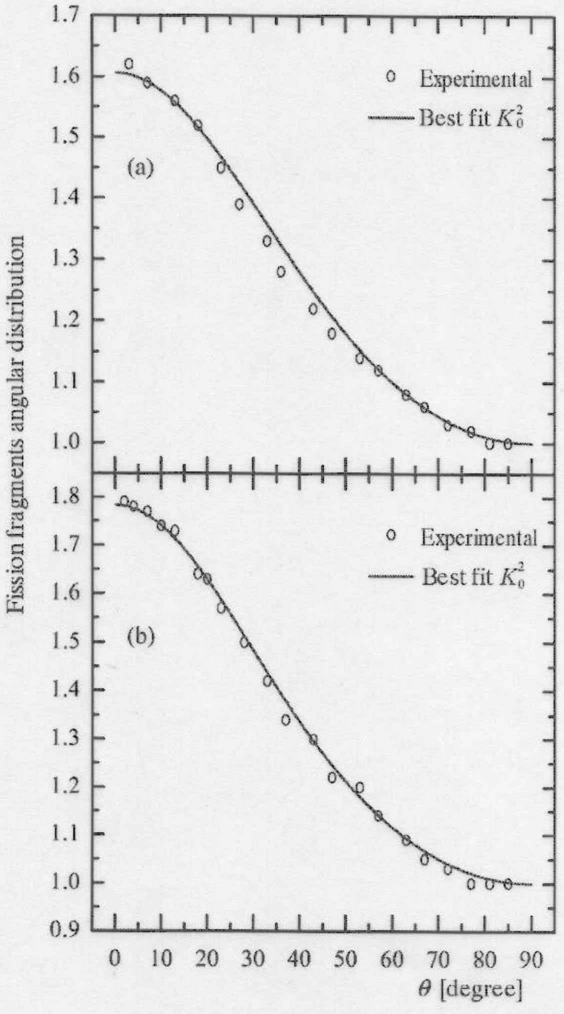

Figure 1. Fission fragment angular distribution for (a) α + ^{232}Th $\to f$ at E_α = 30 MeV and (b) α + ^{232}Th $\to f$ at E_α = 42 MeV

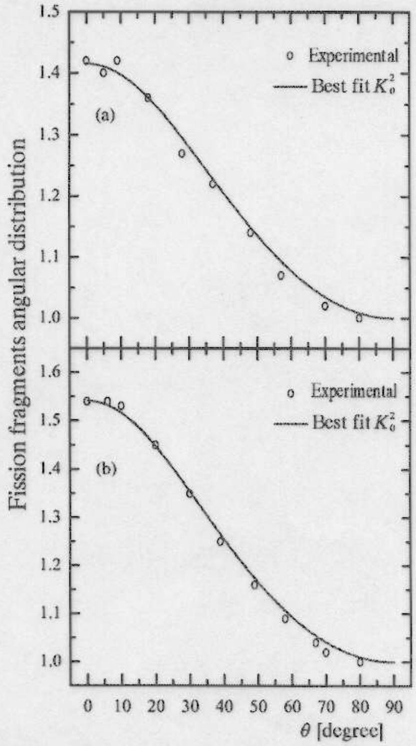

Figure 2. Fission fragment angular distribution for (a) α + ^{238}U $\rightarrow f$ at E_α = 37 MeV and (b) α + ^{238}U $\rightarrow$ f at E_α = 43 MeV

Table 1. Calculated parameters for α + ^{232}Th $\rightarrow f$, α + ^{233}U $\rightarrow f$, and α + ^{238}U $\rightarrow f$ reactions

E_α [MeV]	W(0) / W(90) (Experimental)	K_0^2 (or S_0^2) (Best fit)	S_0^2 (Theory)
	α + ^{232}Th $\rightarrow f$		
16.0	1.09	219.04	51.48
16.8	1.13	146.41	52.74
17.3	1.12	190.44	53.48
18.4	1.34	70.52	55.13
19.7	1.32	82.81	57.00
21.1	1.27	108.16	58.97
21.9	1.29	114.49	60.04
22.9	1.30	104.04	61.39
24.8	1.52	68.06	63.84
26.9	1.70	53.29	66.47
30.0	1.63	71.40	70.16
42.0	1.54	82.81	82.86
	α + ^{233}U $\rightarrow f$		
18.2	1.10	275.89	112.16
20.0	1.11	272.25	115.33
21.1	1.15	211.70	117.25
22.1	1.12	295.84	118.94
23.4	1.80	216.09	121.13
25.3	1.90	218.15	124.24
27.2	1.24	155.75	127.30
28.9	1.30	145.68	129.94
	α + ^{238}U $\rightarrow f$		
17.3	1.80	139.24	89.42
18.2	1.14	163.84	90.49
19.7	1.23	123.21	92.22
21.1	1.21	156.25	93.80
23.2	1.19	161.29	96.15
25.0	1.32	121.00	98.10
27.1	1.38	108.78	100.34
28.9	1.41	104.04	102.23
37.0	1.42	127.46	110.30
43.0	1.54	119.24	116.00

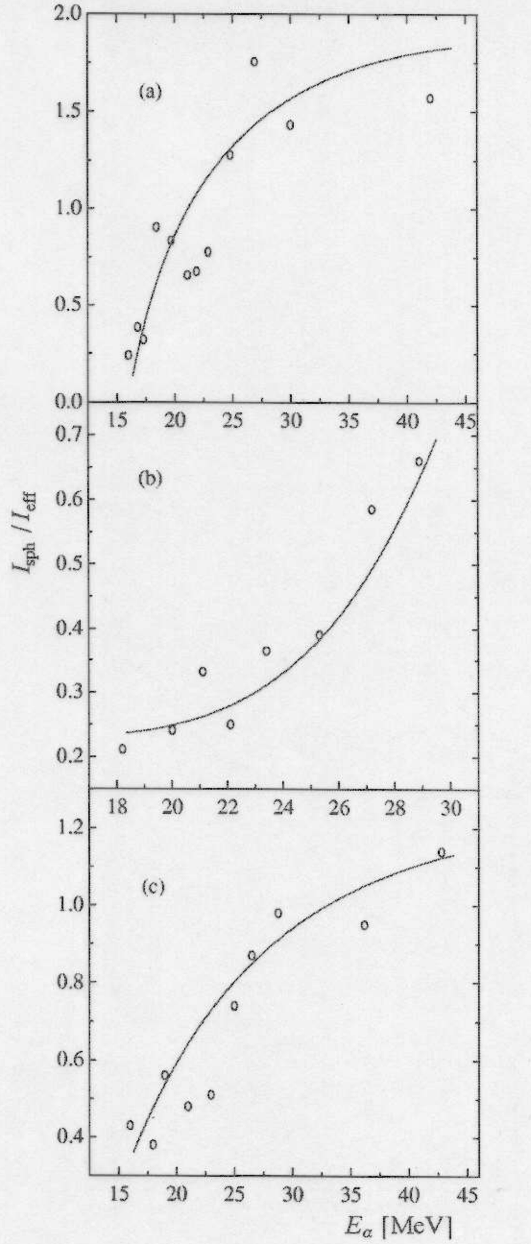

Figure 3. The plotted ratio of I_{sph}/I_{eff} for fissioning nucleuse (a) ^{236}U (b) ^{237}Pu (c) ^{242}Pu

tions of fragments with eq. (5) or (7), and for ^{233}U(5/2$^+$) with eq. (6). The results in the 30 MeV and 42 MeV alpha induced fission of ^{232}Th and in the 37 MeV and 43 MeV of ^{238}U are displayed in figs. 1 and 2. From figs. 1 and 2 it can be seen that the agreement between our theoretical calculations and their corresponding experimental values is quite good. The similar results are shown in the third column of tab. 1.

With consideration of deformed fragments at the scission configuration, the variances S_0^2 have also been calculated by using eq. (10). The computed theoretical values of S_0^2 with the assumed values r_0 = 1.255 fm and E_{def} = 5 MeV, LDP = A/8 and E_{def} = 10 MeV, LDP = A/20 and E_{def} = 5 MeV,

$LDP = A/10$, respectively, for alpha induced fission reactions of ^{232}Th, ^{233}U, and ^{238}U are listed in the last column of tab. 1. From tab. 1 it can be seen that by increase of alpha energy the difference between best fitted values and theoretical results of S_0^2 is decreasing. From this behavior we have concluded that, in general, the variance S_0^2 produced by the SSM for experimental angular distribution of fission fragments in alpha reactions shows a good agreement with the theoretical calculation in higher energies. This establishes the applicability of this model in higher energies and for systems with well-defined deformations, but the TSM shows higher performance in low energies for explanation of the angular distributions of fission fragments in alpha induced fission reactions.

The calculated values of K_0^2 have also been used to compute the characteristics of a nucleus undergoing the fission process. The ratios of I_{sph}/I_{eff} for ^{236}U, ^{237}Pu, and ^{242}Pu are plotted in fig. 3. The examination of these figures shows monotonic dependence on the characteristics of the nucleus undergoing the fission process.

REFERENCES

[1] Rossner, H., Huizenga, J. R., Schorder, W. U., Fission Fragment Angular Distribution, *Phys. Rev. C, 33* (1986), 2, pp. 557-575

[2] Bond, P. D., Fission Fragment Angular Distribution, *Phys. Rev. Lett., 52* (1984), pp. 414-416

[3] Vandenboch R., Huizenga, J. R., Nuclear Fission, Academic Press, New York and London, 1973, p. 109

[4] Ramamurthv, V. S., Kapoor, S. S., Interpretation of Fission Fragment Angular Distributions in Heavy-Ion Fusion Reactions, *Phys. Rev. Lett., 54* (1985), pp. 178-181

[5] Huizenga, J. R., *et al.*: Note on Interpretation of Fission-Fragment Angular Distributions at Moderate Excitation Energies, *Phys. Rev., 174* (1968), 4, pp. 1539-1544

[6] Vaz, L. C., Alexander, J. M., Fussion Barriers Empirical and Theoretical: Evidence for Dynamic Deformation in Subbarrier Fusion, *Phys. Rep., 69* (1981), 4, pp. 375-399

[7] Chaudhry, R., Vandenbosch, R., Huizenga, J. R., Fission Fragment Angular Distributions and Saddle Deformations, *Phys, Rev., 126* (1962), pp. 220-227

[8] Huizenga, J. R., Behkami, A. N., Meadows, J. W., Klema, E. D., Nuclear Pairing Energy of Transition Nucleus ^{240}Pu, *Phys. Rev., 174* (1968), pp. 1539-1544

[9] Behkami, A. N., Roberts, J. H., Loveland, W., Huizenga, J. R., Structure of the Fission Transition Nucleus ^{235}U, *Phys. Rev., 171* (1968), pp. 1267-1277

[10] Griffin, J. J., Fission Fragment Angular Distributions by Exact Power Series, *Phys. Rev., 197* (1962), 4, pp. 1248-1255

[11] Huizenga, J. R., Schroder, W. U., Rossener, H., *Proceedings*, International Symposium on Heavy-Ion Physics (Eds. S. Kubano, M. Ishihara, M. Ichimura), *J. Phys. Soc. Jpn. Suppl., 54* (1985), pp. 257-264

[12] Behkami, A. N., Nazazadeh, P., Fission Fragment Angular Anisotropies and Inertia Parameters, *J. Sci. Iran, 9* (1998), 2, pp. 124-129

[13] Vaz, L. C., Alexander, J. M., Reassessment of Fission Fragment Angular Distributions from Continum States in the Context of Transition-State Theory, *Phys. Rep., 97* (1983), 1, pp. 1-30

[14] Behkami, A. N., Karegar, Z., Nazarzadeh, P., Nasrabadi, M. N., Microscopic Study of Fission Fragment Angular Distributions, *Phys. Rev. C, 63* (2001), 6, pp. 064607-1 – 064607-5

[15] Datta, T., Dange, S. P., Naik, H., Manohar, S. B., Angular Distribution in Alpha-Induced Fission of ^{232}Th and ^{238}U, *Phys. Rev. C., 48* (1993), 1, pp. 221-227

[16] Eremenko, D. O., *et al., Proceedings*, Conference on Nuclear Data for Science and Technology (Eds. G. Reffo, A. Ventura, C. Grandi), Vol. 59, SIF, Bologna, Italy, 1997

Фархад М. РАХИМИ, Омид Н. ГХОДСИ, Азиз Н. БЕХКАМИ

УГАОНА РАСПОДЕЛА И ПАРАМЕТРИ ИНЕРЦИЈЕ ПРИ АЛФА ИНДУКОВАНОЈ ФИСИЈИ ^{232}Th, ^{233}U и ^{238}U

Анализа угаоне расподеле изабраног фисионог фрагмента насталог индукованом фисијом извршена је коришћењем егзактног теоријског израза. Модовима прелазног стања теоријски одређене анизотропије упоређене су са одговарајућим експерименталним вредностима. Сагласност израчунатих и експерименталних резултата је врло добра. Вредности статистичког параметра коришћене су за израчунавање параметра инерције, а добијени резултати упућују на пораст момента инерција.