THE PHYSICAL BASIS OF THE FISSION WAVE REACTOR

by

Volodymyr M. PAVLOVYCH[1], Volodymyr M. KHOTYAYINTSEV[2], and Olena M. KHOTYAYINTSEVA[1]

Received on September 22, 2008; accepted on October 22, 2008

The main idea of slow nuclear fission wave reactor is discussed and short review of the existing works is also presented. The aim of this paper is to clarify the physics of processes, which define the stationary wave of nuclear burning, and to develop the approaches determining the wave parameters. It is shown that the diffusion equation for fluence can be used to describe the stationary and non-stationary processes in the nuclear fission wave. Two conditions of stationary wave existence are first formulated in the paper. The rule of determination of wave velocity as the eigenvalue of boundary problem is also formulated.

Key words: reactor, fission, autowave, nuclear burning

INTRODUCTION

The idea of creating a fast reactor which could, due to slow fission wave propagation, operate over a long period of time without the participation of any staff was first suggested by L. P. Feoktistov [1]. The idea is simple and beautiful. Let us imagine a cylinder of pure fertile material such as ^{238}U or ^{232}Th whose butt-end is irradiated by neutrons. The fertile material is transmuted into fissile material in the superficial region, determined by the path lengths of the neutrons, according to the well known chains of transformations

$$^{238}_{92}\mathrm{U} \; ^{1}_{0}\mathrm{n} \; ^{239}_{92}\mathrm{U} \xrightarrow[\beta,\, T_{1/2}\ 23.5\ \mathrm{min}]{} ^{239}_{93}\mathrm{Np} \xrightarrow[\beta,\, T_{1/2}\ 2.35\mathrm{d}]{} ^{239}_{94}\mathrm{Pu}$$

$$^{232}_{90}\mathrm{Th} \; ^{1}_{0}\mathrm{n} \; ^{233}_{90}\mathrm{Th} \xrightarrow[\beta,\, T_{1/2}\ 22\ \mathrm{min}]{} ^{233}_{91}\mathrm{Pa} \xrightarrow[\beta,\, T_{1/2}\ 27\ \mathrm{d}]{} ^{233}_{92}\mathrm{U}$$

Nuclear Technology & Radiation Protection
Scientific paper
UDC: 621.039.524.46
BIBLID: 1451-3994, *23* (2008), 2, pp. 3-15
DOI: 10.2298/NTRP0802003P

Authors' addresses:
[1] Kyiv Institute for Nuclear Research,
National Academy of Sciences of Ukraine
47, Prospekt Nauki, 03680 Kyiv, Ukraine
[2] Kyiv National University, Physical Faculty
64, Volodymyrska Str., 01033 Kyiv, Ukraine

E-mail address of corresponding author:
pavlovich@kinr.kiev.ua (V. M. Pavlovych)

When the critical concentration of fissile material is reached, the self-sustained chain reaction begins and neutrons are supplied to the neighbouring region where the fissile material is accumulated, and so on. Thus the slow wave of nuclear fission propagates through the fertile material. Such a wave is self-regulated, because any fluctuation exceeding the critical concentration of fissile material should burn up during the neutron's lifetime, while the new fissile material is formatted in times compared with the time of precursor β-decay, not simultaneously.

One important condition was established by Feoktistov: the critical concentration of fissile material should be lower than its equilibrium concentration (which is determined as asymptotic at $t \to \infty$ solution of the kinetic equation for the concentration of fissile material). So, the propagation of the nuclear fission wave, in front of which the uranium captures the neutrons and turns into plutonium, is possible in the ^{238}U medium. This is a typical auto-wave regime which is extensively studied in non-linear physics.

Later E. Teller [2] has proposed a fast reactor with a thorium cycle which operates in a self-aligning regime, at a depth of 100 m underground, over a period of 30 years, without human interference. Actually, Teller has used Feoktistov's idea, applied to the thorium cycle.

In the works by Goldin *et al.* [3, 4], the self-aligning regime of a fast reactor was studied on the basis of the differential equation system which represents the cylindrical reactor with an inner critical core and an outer ^{238}U blanket. Such a reactor can op-

erate without external control over a long period of time, but the auto-wave regime does not appear and reactor power first slowly increases and then decays after a year of operation. In short, this reactor operates in a non-steady-state regime.

The steady-state operation of slow fission waves was studied by several groups. Sekimoto *et al.* [5-7] performed a numerical modeling of different types of fast reactors in a many group diffusion approximation which took into account spatial and temperature effects, but details of these calculations were not published. The authors named the steady-state fission auto-wave strategy for the development of fast reactors CANDLE. It was also shown in [8, 9] that, with the help of numerical modeling, in a two-zone homogeneous reactor with metallic U-Pu fuel, Na coolant and Fe as construction material, the wave regime of nuclear burning, which can exist over a long period of time, can be obtained (flat, one-dimensional model). Rusov *et al*. [10] suggested that such a reactor could operate inside the Earth, at the border between the solid and liquid core; they gave physical arguments for the existence of a wave reactor and, on the basis of 3-D numerical modeling, presented evaluations of the power of such an Earth reactor.

In our opinion, this type of numerical calculations do not reflect the physical peculiarities of wave reactors and, in any case, are not to be considered a proof of the existence of stable waves. This is due to the fact that the principal idea of the wave reactor's self-aligning is lost in such calculations: fluctuation excesses of plutonium concentration over the critical value burn up over a neutron's lifetime (without taking the delayed neutrons into consideration) or, at least, in comparison to the reactor's period (taking the delayed neutrons into account), but new plutonium formats approximately three days later, not simultaneously. This means that numerical calculations should be performed with a time mesh of the order of 10^{-6}-10^{-7} s, without delayed neutrons and in time steps of 0.1-1 s, taking the delayed neutrons into account. At the same time, most numerical calculations were performed with time steps of the order of days (in studies where this was indicated) because of the necessity to calculate the long-term processes of wave formation and propagation over the whole reactor. But the calculation with such time steps leads to a seeming increase in the multiplication factor of up to 1.1-1.2 in, depending on the time step, while reactor runaway is not observed. These mean that such calculations have no physical sense.

It is interesting to note that, at first glance, taking into account the delayed neutrons is not crucial for the formation of the nuclear burning wave. But it may play an important role in its numerical calculation, since the adding of one or several equations to the mathematical model cannot be compared to the several order decrease in the necessity time step. It is clear that the calculation during a reactor lifetime at small time steps of the neutron lifetime order is impossible for existing computers (excluding supercomputers and international grids).

Analytical studies of the processes of wave formation are based on simplified equations with following numerical evaluations. In particular, in the interesting works by Van Damm [11,12] (see also [13] where on the basis of Van Damm's model the account for a backling was performed), auto-wave ignition and formation in the one-dimensional system on the basis of the one-group diffusion equation with non-linear reactivity feedback and model dependence of the multiplication factor on fluence was studied. This dependence, approximately, in an implicit form, accounts for the kinetic burning equations, in particular the accumulation of plutonium. As a result, the steady-state solution in the form of a diffusion solitone is obtained and, also, the process of auto-wave ignition is numerically studied.

In our previous work [14], we have studied the properties of the stationary neutron fission wave depending on neutron-nuclei system parameters in a model close to the Feoktistov's one. We have shown that the description of the system simplifies in the cases of fast and slow waves. The critical concentration of plutonium was considered as a given constant, although in reality it changes during wave propagation. In this work we excluded this drawback also taking into account other nuclides which are formed as a result of wave propagation. The real control parameter of the system – the external absorber concentration – is also introduced. Changing it, one can obtain the desired velocity of wave propagation.

NUCLEAR BURNING WAVE IN THE ONE-DIMENSIONAL APPROXIMATION: FLUENCE DIFFUSION EQUATION

The usual breeder reactor (BR) and nuclear fission wave reactor (NFWR) are basically systems of interacting neutrons and nuclei. Nuclides ^{238}U and ^{232}Th can capture neutrons and transform them to ^{239}Pu and ^{233}U, correspondingly. The characteristic process for both reactor types is the transformation of a fertile nuclide to a fissile nuclide due to neutron capture. Therefore, the theory of BR and NFWR should be formulated by means of similar equations. These are differential equations which describe the time variation of nuclide concentration due to nuclide transformation chains (burn up equations) and the neutron transport equation taking into account their spatial migration, energy change, and interaction with nuclei.

The simplest approach uses the one-group approximation for neutron energy distribution and diffusion approximation for spatial transport. Such an equation has the form

$$\frac{\partial \phi}{\partial t} \quad vD\Delta\phi \quad g\phi \tag{1}$$

where $\phi \quad vn$ is the neutron flux density, v – the mean neutron velocity, n – the neutron concentration, D – the diffusion coefficient, and g – the neutron generation function which is the linear function of nuclei concentrations N_i. Equation (1) is the balance equation taking into account the change in the number of neutrons in the elementary volume due to diffusion, absorption by nuclei, and creation by fission. We did not take into account the delayed neutrons, deeming them insufficient in the wave propagation process.

The specificity of the model is defined by the level of detail of the nucleus kinetic model and the explicit form of neutron generation function $g(N_i)$. First of all, we shall consider the general consequences of eq. (1) and then we shall show the effect of the model detailes.

It is necessary to note that there are stationary and non-stationary processes concerning NFWR. The steady-state regime for a fission wave reactor is the regime of stationary wave propagation, and the ignition and formation of stationary waves are sufficiently non-stationary processes. Let us show that one can use the diffusion equation for fluence to describe both stationary and non-stationary processes in NFWR. This opens new possibilities for analytical studies of the system.

Let us introduce the fluence field as the time integral of the neutron flux density field

$$\psi(\vec{r},t) \quad \int_{t_0}^{t} \phi(\vec{r},\tau)\mathrm{d}\tau \tag{2}$$

where t_0 is the initial moment of time. Equation (2) is equivalent to the following differential equation with zero initial condition

$$\frac{\partial \psi}{\partial t} \quad \phi(\vec{r},t), \quad \psi(\vec{r},t_0) \quad 0 \tag{3}$$

In general, let us include into the right hand side of eq. (1) the term $s(\vec{r},t)$ describing the external neutron source. Then, integrating eq. (1) in time, we obtain the equation for fluence which also has the form of the diffusion equation

$$\frac{\partial \psi}{\partial t} \quad vD\Delta\Psi \quad G \quad S(\vec{r},t) \tag{4}$$

Here S is the fluence source which also depends on the initial value of neutron flux density

$$S \quad \int_{t_0}^{t} s(\vec{r},t)\mathrm{d}\tau \quad \phi(\vec{r},t_0) \tag{5}$$

And, let us name value G, fluence generation function

$$G \quad \int_{0}^{\psi} g(\vec{N})\mathrm{d}\psi \tag{6}$$

It is defined by the full number of neutrons created in the elementary volume during the entire process, from t_0 to t, taking into account absorption. Equation (6) should be understand as an integral along the system trajectory in the space of variables $(\vec{N},\psi)$ for the given point of volume. For NFWR, such a trajectory is fully defined by the kinetic equation for nuclear concentration with initial conditions, if the dependence ψ on time is definite in the given point. Besides

$$\frac{\partial G}{\partial t} \quad g\phi \tag{7}$$

As it is shown in the following two sections, one can express the neutron number increment through the increment of the number of nuclei which take part or are created in corresponding processes. Thus, G is the function of initial nuclei concentrations, final concentrations, and fluence, and it does not depend on the trajectory of the transition from the initial to the final state

$$G \quad G(\vec{N}_0,\vec{N},\psi) \tag{8}$$

The said fact gives an informal sense to eq. (4) and permits us to use it in the study of non-stationary processes in NFWR. Function G in the explicit form is constructed and analyzed in the last two sections for two definite models of nucleus kinetics.

We shall restrict ourselves to the consideration of a one-dimensional model only since the issue of transverse and longitudinal leakages constitutes a separate problem and will be considered elsewhere.

GENERAL PROPERTIES OF THE STATIONARY WAVE PROBLEM

Let the external source be absent, $s = 0$, the wave coming from infinity, the initial moment equaling $t_0 = -\infty$, and neutrons at the initial moment at the finite point being absent. In this case, the NFWR may be described by a system of equations which includes eq. (6) at $S = 0$ and nucleus kinetic equations of the following general form

$$\frac{\partial \psi}{\partial t} \quad vD\Delta\psi \quad G(\vec{N}_0,\vec{N},\psi) \tag{9}$$

$$\frac{\partial \vec{N}}{\partial t} \quad \hat{\sigma}\vec{N}\frac{\partial \psi}{\partial t} \quad \hat{\lambda}\vec{N} \tag{10}$$

where $\hat{\sigma}$ is the matrix, elements of which are the microscopic cross-sections of absorption and capture, and $\hat{\lambda}$ the matrix, elements of which are the decay constants of β-active nuclides.

Let the wave propagate in the infinity medium along the $0x$ axis, from right to left. For a stationary wave of velocity u, all the fields are functions of one wave variable $x + ut$, whose value varies from $-\infty$ to $+\infty$. Let T be the characteristic time of wave propaga-

tion and L be its characteristic size. Then, one can define the dimensionless wave variable as

$$z \quad \frac{x}{L} \quad \frac{t}{T} \tag{11}$$

We shall define the value of L below, and consider T as a free parameter which determines wave velocity as $u = = L/T$. So that the dependencies of all the fields on the dimensionless coordinate x/L and dimensionless time t/T are actually the same and coincide with the dependence on the wave variable z. The system of partial derivatives (9)-(10) is transformed to the system of ordinary differential equations of the wave variable z.

The processes which take place during wave propagation are characterized by several, sufficiently different time scales. In addition to T, other time scales are determined by the neutron life time τ. The mentioned life time is connected to the function g, and can be properly defined in the definite model only. It is important that $g\tau$ is the value of the order of unity and therefore $G\tau \quad \psi$. Besides, it is known that in fast reactors the neutron lifetime of the order of $\tau \quad 10^{-6}$ -10^{-7}s, while T varies from days to months, depending on reactor power. So $\varepsilon = \tau/T \ll 10^{-7}$ is an extremely small parameter. Another time scale defines the β-decay of the intermediate nuclide, τ_β, which, in order of magnitude, could be compared with T, but is really always $t_\beta < T$.

Let us define the following values in order to write the equations in dimensionless form

$$\hat{\sigma} \quad \frac{\sigma}{\sigma_1}, \ \psi \quad \sigma_1\psi, \ g \quad \tau g,$$
$$G \quad \sigma_1\tau G, \ \vec{N} \quad \frac{\vec{N}}{N_{10}} \tag{12}$$

where σ_1 and N_{10} are the absorption cross-section and initial concentration of the fertile nuclide, correspondingly, and dimensionless values are denoted by primes which are omitted below. The values $\hat{\lambda}$ and T are still dimensional, but the product $T\hat{\lambda}$ is dimensionless.

Multiplying eq. (9) by τ, one can obtain

$$\varepsilon\dot{\psi} \quad v\frac{D\tau}{L^2}\ddot{\psi} \quad G(\vec{N},\psi) \tag{13}$$

$$\dot{\vec{N}} \quad \dot{\psi}\hat{\sigma}\vec{N} \quad T\hat{\lambda}\vec{N} \tag{14}$$

where z derivatives are denoted by dots. Since ε is extremely small, the term $\varepsilon\dot{\psi}$ in eq. (13) can be neglected with a high precision (the neutron diffusion process is actually stationary). Then it is seen from eq. (13) that one can define L as the length of neutron diffusion displacement during lifetime τ

$$L^2 \quad vD\tau \tag{15}$$

As a result, the system (13)-(14) takes the form

$$\ddot{\psi} \quad G(\vec{N},\psi) \tag{16}$$

$$\dot{\vec{N}} \quad \dot{\psi}\hat{\sigma}\vec{N} \quad T\hat{\lambda}\vec{N} \tag{17}$$

The stationary wave is a partial solution of the system (16)-(17) and should satisfy some additional conditions. Since the correct mathematical formulation of the problem is not obvious, it is necessary to take into account certain physical considerations.

The stationary burning wave is the transition of the system from one homogeneous steady-state to another. This initial state, the state of "fuel", is a given one. It is established as

$$N_i(x,t_0) \quad N_{i0,} \quad n(x,t_0) \quad 0, \ \psi \quad 0 \tag{18}$$

The final state, state of "ashes", is to be determined in the process of solving the problem.

$$N_i(x,t_\mathrm{f}) \quad N_{\mathrm{if},} \quad n(x,t_\mathrm{f}) \quad 0, \ \psi \quad \psi_\mathrm{f} \tag{19}$$

Both states are the solutions of the system (9)-(10) in variables x, t, and the solutions of the system (16)-(17) is relative to the wave variable z too.

As the initial and final states are stationary, the initial and final concentrations of all unstable nuclei should be equal to zero $N_{i0} = N_{\mathrm{if}} = 0$. In reality, this restriction concerns nuclei with a relatively small lifetime of β-decay, $\tau_{\beta i} \quad T$. In addition, both states are obviously subcritical, therefore $g(N_{i0}) < 0$ and $g(N_{\mathrm{if}}) < 0$.

The initial and final states are the stationary points to which the solution of the system (16)-(17) should tend at $z \quad \infty$ and $z \quad +\infty$. The linearization of eq. (16) with respect to small deviations from the steady-states (18)-(19) leads to linear equations of the stationary diffusion of the same form

$$\frac{\mathrm{d}^2\tilde{\psi}}{\mathrm{d}z^2} \quad \alpha^2\tilde{\psi} \quad 0 \tag{20}$$

with two exponential partial solutions $\tilde{\psi} \quad \mathrm{e}^{\alpha z}$. Here $\tilde{\psi} \quad \psi, \ \alpha = \alpha_0$ for the initial state, and $\tilde{\psi} \quad \psi_\mathrm{f} \quad \psi, \alpha = = \alpha_\mathrm{f}$ for the final state, the constants being defined by nuclei concentrations in "fuel" and "ashes"

$$\alpha_0^2 \quad g(\vec{N}_0) \quad 0, \quad \alpha_\mathrm{f}^2 \quad g(\vec{N}_\mathrm{f}) \quad 0 \tag{21}$$

Rise-up and descending parts of the wave are described by the exponentially decreasing functions at $z \quad -\infty$ i $z \quad +\infty$, correspondingly

$$\psi(z) \quad A\mathrm{e}^{\alpha_0 z} \quad A\mathrm{e}^{\frac{x \ ut}{L_0}}$$
$$\psi_\mathrm{f} \quad \psi(z) \quad B\mathrm{e}^{\alpha_\mathrm{f} z} \quad B\mathrm{e}^{\frac{x \ ut}{L_\mathrm{f}}} \tag{22}$$

where $L_0 = L/\alpha_0$ and $L_\mathrm{f} = L/\alpha_\mathrm{f}$.

Thus, the solution of eq. (20) should satisfy the following conditions

$$z \quad \infty: N_i \quad N_{i0}, \ \psi \quad 0, \ \dot{\psi} \quad 0, \ddot{\psi} \quad 0, \ldots \tag{23}$$

$$z \quad \infty: N_i \quad N_{\mathrm{if}}, \ \psi \quad \psi_\mathrm{f}, \ \dot{\psi} \quad 0, \ddot{\psi} \quad 0, \ldots \tag{24}$$

Among them, independent conditions which do not contain unknown values are the following

$$z \quad \infty: N_i \quad N_{i0}, \ \psi \quad 0 \qquad (25)$$

$$z \quad \infty: \dot{\psi} \quad 0 \qquad (26)$$

So, the formulation of the problem includes equations (16) and (17), boundary conditions (25) and (26) and the condition of neutron flux density non-negativity

$$\dot{\psi} \quad 0 \qquad (27)$$

Seemingly, the number of conditions are equal to the order of the system (16)-(17): there are first order equations for every N_i with initial conditions and two conditions to the second order eq. (16). But, it can be seen that the problem (16,17)-(25, 26, 27) does not change with the replacement $z \quad z–C$ which reflects the obvious demand: the location of the wave at the wave coordinate is arbitrary. So, one of the conditions (25)-(27) is redundant, as should be in an eigenvalue problem.

The problem (16,17)-(25, 26, 27) always has a trivial solution which corresponds to its initial state. In this solution, the initial and final states coincide. The non-trivial solution exists if one satisfies problem conditions due to the free parameter which is called the spectral parameter of the problem. In the given case, such a parameter is *T*. This parameter determines the velocity of the wave $u = L/T$. Thus, one can obtain the stationary wave field distribution integrating the system (16)-(17) with the known *T*. So, in the given physical system, the velocity of the stationary wave has a definite value, if such a wave exists.

But the task of NFBW theory is not only the calculation of wave velocity and other wave characteristics. It is also necessary to prove the possibility of wave formation at definite real initial conditions, and the solution of the problem of the stationary wave has a physical meaning only in the case when it offers a stable solution for the non-steady system (13)-(14). This issue surpasses the framework of our paper.

NFWR as a physical system and the corresponding mathematical model can be characterized by a set of material parameters which can be changed practically. Such independent parameters are called control parameters of the system. Their numerical values define the point in the control parameter space. The problem eigenvalue is obviously a function of control parameters.

The existence of control parameters is very important because in a non-linear system the eigenvalue can be absent or there can be several eigenvalues, depending on the values of the control parameters. In non-linear systems, a situation when the solution at $z \quad \infty$ tends to different bistability waves, depending on initial conditions, is also possible. Thus, practically the most important aim of the theory is the study of the eigenvalue spectrum and wave characteristics dependent on point location in the control parameter space and the provision of optimal/prescribed characteristics of the NFWR.

THE GENERAL CONDITIONS OF WAVE STATIONARITY

The boundary problem (16), (17-25), (26, 27) can not be generally solved analytically, but one can find some general conditions for the stationary wave. The configuration space for the system (16)-(17) is the space of variables (N_i, ψ), while the phase space is the space of variables (N_i, ψ, $\dot{\psi}$). When the system moves from the initial to the final state, fluence ψ monotonously varies with coordinate *z* from zero to ψ_f, $0 \quad \psi \quad \psi_f$. Therefore, one can consider that $\vec{N}$ and $\dot{\psi}$ are the functions of ψ at phase trajectory. Let us multiply the eq. (16) by $\dot{\psi}$ and integrate it from the initial conditions (25) to the current state. Taking into account that in the initial state $\dot{\psi} \quad 0$, for the current state, one can obtain

$$\frac{1}{2}\dot{\psi}^2 \quad M(\vec{N},\psi) \quad 0 \qquad (28)$$

$$M(\vec{N},\psi) \quad \int_0^{\psi} G(\vec{N},\psi)\mathrm{d}\psi \qquad (29)$$

Thus, in order to satisfied the boundary conditions in the final state (24) $\dot{\psi} \quad 0$, $\ddot{\psi} \quad 0$ at $z \quad \infty$, it is necessary and sufficient to satisfy to the following two conditions

$$G(\vec{N}_f,\psi_f) \quad 0 \qquad (30)$$

$$M(\vec{N}_f,\psi_f) \quad 0 \qquad (31)$$

where the integral is taken along the trajectory of the system $\vec{N} \quad \vec{N}(z)$, $\dot{\psi} \quad \psi(z)$. These are just the general conditions defining the stationary wave. More exactly, the conditions (30) and (31) define two parameters – wave velocity (or parameter *T*) and final fluence. The solution of (16) and (17) satisfying the boundary conditions (25-27) with these parameters corresponds to the stationary wave, *i. e.* satisfies all problem conditions.

It is interesting to note that the condition $\ddot{\psi} \quad 0$ at $z \quad +\infty$ is the effect of condition $\dot{\psi} \quad 0$(26), but conditions (30, 31) are really different and are both necessary. Independent arguments can be presented to prove this. Equation (16) has the form of a Newton equation for the particle of a unit mass with coordinate ψ, which moves under the influence of force $F = –G$. The particle begins the movement at the moment $z = –\infty$, without initial velocity from point $\psi = 0$, in which $\dot{\psi} \quad 0$ also, so the initial point is the equilibrium position. At first, the particle gathers speed under the influence (at first of a positive and then of a negative force), then gradually inhibits and at $z = +\infty$ reaches the final point $\psi = \psi_f$, where it stops. This is possible if $\psi = \psi_f$ is also

the state of equilibrium where $G = 0$ – condition (30), and the work of force G during the displacement 0 ψ_f is equal to zero – condition (31), because the kinetic energy at initial and final points is zero.

If at some point $\psi = \psi_f$ force G becomes zero, but the speed persists to be positive, the particle does not stop and displaces in the region $\psi = \psi_f$. If at some point the speed, which was positive, becomes zero and the force (negative) is not zero, the acceleration is still negative and the speed changes sign: the particle passes the turning point. Such a trajectory also has no physical sense, since function $\dot{\psi}$ is proportional to the neutron flux and always $\dot{\psi}$ 0. But this mechanical analogy has limits, because the problem with arbitrary initial conditions has to have a physical sense in mechanics. In our case, the boundary problem of steady diffusion in which the conditions are fixed at $z = \pm\infty$ is considered. Therefore, phase trajectories which do not satisfy the boundary condition at $z = \pm\infty$ have no physical sense (see fig. 1).

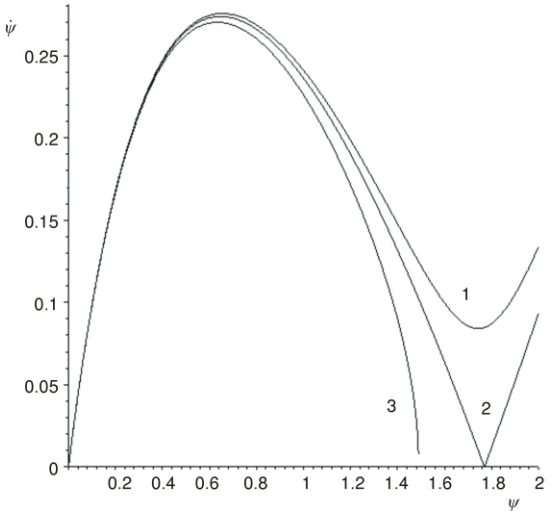

Figure 1. The phase trajectories in the plane (ψ, $\dot{\psi}$) for defferent values of absorber concentration p in the limite W 0; for curve 1, $p > p_0$; for curve 3, $p < p_0$; for curve 2, $p = p_0$

Generally, conditions (30) and (31) are the conditions of equilibrium which provide for wave stationarity. They reflect the integral space-time properties of the system and are the analogies of criticality conditions in usual reactors. But, the essential difference, when NFRW are concerned, is that at least two of such conditions are present.

The physical meaning of condition (30) is obvious for steady diffusion: this is an integral condition of neutron generation and absorption equality along the wave. Contrary to condition (31), only its geometrical meaning is obvious: the area under the curve $G(\psi)$ along the trajectory should be equal to zero (the area is positive, where $G(\psi)$ 0, and negative, where $G(\psi)$ 0, see fig. 2). All that remains to be said is that it re-

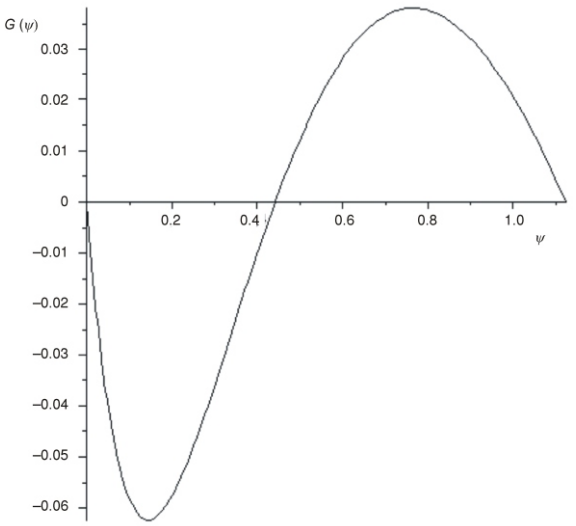

Figure 2. The dependence of the fluence generation function on the dimensionles fluence

flects the distribution of neutron production along the fluence coordinate.

Let us write the wave stationarity conditions in terms of the neutron generation function

$$\int_0^{\psi_f} g\,d\psi = 0 \qquad (32)$$

$$\int_0^{v} (\psi_f \quad \psi)\, g\,d\psi \quad 0 \qquad (33)$$

where g is the function of the fluence along the system trajectory. Obtained conditions coincide with the conditions of level static equilibrium under the influence of perpendicular to the level distributed force $g(\psi)$, which is acting at the interval 0 ψ ψ_f(ψ is the coordinate along the level). Conditions (32) and (33) are the conditions of the sum of the forces and sum of the force moments equal to zero, correspondingly. Therefore, the first condition is the integral condition of neutron creation and absorption equality, and we can call the second condition the integral condition of neutron creation and absorption "moments" equality. Such a momentum is calculated in (33) relative to the point $\psi = \psi_f$. Of course, if the condition (32) is fulfilled, the momentum can be calculated relative to the arbitrary point. We shall call (below) function M – neutron generation momentum.

One can transform conditions (32) and (33) to those analogous to the criticality condition of usual reactors: $k_{eff} = 1$, where k_{eff} is the effective multiplication factor. Let us, for this purpose, disjoint the neutron and fluence generation functions into two parts

$$g \quad g \quad g \qquad (34)$$

so that $\bar{g}$ and $\bar{g}$ are positive (the bars denote the mean values). Besides, coresponds to neutron genera-

tion and coresponds to neutron absorption. Then the effective multiplication factor for a usual reactors has the form

$$k_{\text{eff}} \quad \frac{g}{g} \tag{35}$$

Let us call these points fluence axis with coordinates

$$\overline{\psi} \quad \frac{\int_0^{\psi_f} \psi g \ \mathrm{d}\psi}{\int_0^{\psi_f} g \ \mathrm{d}\psi} \tag{36}$$

"centers of neutron generation and neutron absorption application". And let us introduce the integral neutron multiplication factor for the NFWR

$$k^0 \quad \frac{\overline{g}}{\overline{g}} \tag{37}$$

and integral coefficient of space equilibrium

$$k^1 \quad \frac{\overline{\psi}}{\overline{\psi}} \tag{38}$$

Then one can write the conditions of wave stationarity (32, 33) in the form

$$k^0 \quad k^1 \quad 1 \tag{39}$$

The advantages of such coefficients setting consist in their different behavior with changes of distributions g^+ and g^-. For example, multiplying g^+ by K, we obtain an increase in k^0 but not in k^1. Therefore, k^0 reflects the absolute value of g^+ and g^-, and k^1 shows their special distribution along the fluence.

Both coefficients are the functions of wave propagation time T (or wave velocity $u = L/T$), maximal fluence ψ_f and system control parameters q_s, and two conditions (39) implicitly determine the dependences $T = T(q_s)$ and $\psi_f = \psi_f(q_s)$.

Summing the obtained results, one can make the following qualitative conclusion: in addition to the integral equality of created and absorbing neutrons, the distribution of neutron creation and absorption density along the fluence coordinate is important for wave stationarity. One can write the corresponding condition of time – space equilibrium in the wave as the condition of generation and absorption centers of application coincidence along the fluence coordinate $\overline{\psi} \quad \overline{\psi}$, or $k^1 = 1$. So the wave should be balanced by two (not just one) parameters simultaneously, which is extremely important.

We shell consider below the specific models of the system in which the nuclear burning wave can propagate, taking into account the different nuclides affecting wave propagation.

NFWR WITH ABSORBER

Let us consider a nuclear system which can be separated into two independent subsystems: "fuel" and "absorber". Let us refer to the fuel the fertile nuclide and all other nuclei appurtenant to the transformation chains of fertile nuclide, *i. e.*, which can format by the successive transformations due to the interaction with neutrons, decays, and fission, including fission products. If there is only one fertile nuclide in the initial state, the concentrations of all nuclei appurtenant to the "fuel" are linearly dependent on the initial concentration of fertile nuclide N_{10}.

In order to regulate the neutron balance in the system, let us introduce into the composition of the system other nuclei-absorbing neutrons. This is the absorber itself. But we also refer to the absorber all nuclei which do not belong to the fuel, including those chemically bounded with fertile nuclide, for example, oxygen, construction materials and so on. The existence of the absorber is crucial, since it can control the properties of the system in definite frames.

The neutron generation function in such a system is the sum of two terms

$$g \quad g^{\text{f}}(\vec{N}) \quad \frac{1}{\tau_{\text{A}}} \tag{40}$$

where g^{f} is the fuel part of the generation function linearly dependent on fuel nucleus concentrations N_i, and τ_{A} is the neutron lifetime relative to the absorption by the absorber nuclei, besides $\tau_{\text{A}} = \text{const}$ (absorber does not saturate).

Passing on to the dimensionless neutron generation function, it is necessary to multiply (40) by neutron lifetime τ

$$g \quad g^{\text{f}}(\vec{N}) \quad p$$

where g, g^{f} are dimensionless (primes are omitted), and $p = \tau/\tau_{\text{A}}$ is the dimensionless effective absorber concentration.

Energy release in the wave region increases, approximately, in proportion to its velocity. Therefore, from a technical point of view, the acceptable level of energy release restricts the velocity range to values ranging from months to years for wave propagation time T. The main inner time scale for the NFWR is determined by the time of β-decay of the intermediate nuclide N_2, $\tau_\beta = \tau_2$. This is ^{239}Np with a characteristic time of β-decay τ_2 =3.47 days per uranium cycle, and ^{233}Pa with β-decay half-life τ_2 =36.6 days for the thorium cycle. Therefore, the value of the actual dimensionless wave velocity

$$W \quad \frac{\tau_2}{T} \tag{41}$$

for the uranium cycle is always small. For the thorium cycle $W < 1$, and when T 1 year also $W \ll 1$. So, we will call the wave with a small value of dimensionless velocity a wave of small velocity. The existence of a small parameter in the problem gives us the possibility to put to practical use wave stationary conditions (32,

33) and to analytically develop the perturbation theory or build an iteration scheme of calculation.

For the sake of simplicity, let us consider that the system has one control parameter – a dimensionless effective concentration of absorber p. In that case, wave stationarity conditions give two equations for three values: p, W, ψ_f. Therefore, $W = W(p)$ and $\psi_f = \psi_f(p)$. Thus, we can obtain waves of different velocities changing the control parameter. But, it is convenient for a wave of small velocity to consider that the velocity of the wave is known. Then, from wave stationarity conditions for the given dimensionless velocity W we can find ψ_f and control parameter p at which the wave of given velocity is realized

$$\psi_f \quad \psi_f(W), \quad p \quad p(W) \tag{42}$$

The scheme of problem solution by the convergence method is following. If $W \ll 1$, one can find all nuclide concentrations N_i, values of G, M, and parameters ψ_f and p in the form of power series of W

$$Q(W) \quad Q^{(0)} \quad WQ^{(1)} \quad \ldots \tag{43}$$

where Q is one of the mentioned values. It is necessary to determine which term in the power series is the first to differ from zero at $W \quad 0$ for the given value, and which term is the small correction to the first one. For this purpose, let us define the first three equations of the system (14) and let us consider all other nuclides and fission products as stable

$$\dot{N}_1 \quad \dot{\psi} N_1 \tag{44}$$

$$\dot{N}_2 \quad \alpha_1 \dot{\psi} N_1 \quad \frac{N_2}{W} \quad \gamma_2 \dot{\psi} N_2 \tag{45}$$

$$\dot{N}_3 \quad \frac{N_2}{W} \quad \gamma_3 \dot{\psi} N_3 \tag{46}$$

Here $\alpha_1 \quad \sigma_{c1}/\sigma_{a1}$, $\gamma_2 \quad \sigma_{a2}/\sigma_{a1}$, $\gamma_3 \quad \sigma_{a3}/\sigma_{a1}$, N_1 is the concentration of the fertile nuclide (^{238}U or ^{232}Th), N_3 is the concentration of the main fissile nuclide (^{239}Pu or ^{233}U). It is seen from eq. (45) that at W

0, the concentration of the intermediate nuclide N_2 tends to zero (it decays quickly compared to the time of wave propagation). Besides terms of zero order, W contains the first term in the right hand side of eq. (45) and terms N_2/W in eqs. (45) and (46). All other terms which contain N_2 in all kinetic equations are the terms of the first order in W, and one should neglect them in zero approximation in W. Then we have from eq. (45)

$$\frac{N_2}{W} \quad \alpha_1 \dot{\psi} N_1(\psi) \tag{47}$$

and substituting this in eq. (46), we exclude N_2 from the nuclear kinetic system.

If we denote all sets of nuclides without N_2 as $\vec{N}$ and take it as stable, then one can pass on from z to the new independent variable ψ, $d\psi \quad \dot{\psi} \, dz$. As a result, the system (17) takes the form of a system with constant coefficients

$$\frac{d\vec{N}}{d\psi} \quad \hat{\sigma}\vec{N} \tag{48}$$

So, in zero approximation in W, nuclide concentrations are determined only by the current value of fluence

$$N_i^{(0)} \quad N_i^{(0)}(\psi), \quad i \quad 2 \tag{49}$$

Besides, the equation for N_1 is integrated independently on W

$$N_1 = e^{-\psi}$$

Therefore, the values of G and M, which are the linear functions of nuclear concentrations, are also the functions of fluence in the zero approximation in W, and conditions of wave stationarity (30) and (31) take the form of the equation system for p and ψ_f

$$G \quad G^{(0)}(p, \psi_f) \quad 0 \tag{50}$$

$$M \quad M^{(0)}(p, \psi_f) \quad 0 \tag{51}$$

Let us denote the solution of this system as p_0 and ψ_{f0}, then in zero approximation $p^{(0)} = p_0$, $\psi_f^{(0)} = \psi_{f0}$.

Furthermore, substituting in eq. (30) the formula for neutron generation momentum M in zero approximation at $p = p_0$, one obtains the equation for a system phase trajectory in zero approximation. Returning to the mechanical analogy in previous section, one can say that in zero approximation the forth field G, in which the particle motion occurs, is potential, and eq. (30) has the form of the energy conservation low at zero full energy. From eq. (28) at $p = p_0$, taking into account (12), we have $\dot{\psi}$, *i. e.* dimensionless neutron flux Φ as a function of fluence

$$\dot{\psi} \quad \Phi(\psi) \tag{52}$$

where

$$\Phi(\psi) \quad \sqrt{2M(p_0, \psi)} \tag{53}$$

Dimensional flux from eqs. (3), (12), and (41) is

$$\phi \quad \frac{W\dot{\psi}}{\tau_2} \tag{54}$$

and it is proportional to the wave velocity in this approximation.

Finally, integrating the first order eq. (52) with an arbitrary initial condition

$$z \quad z_1 \quad \int_{\psi_1}^{\psi} \frac{d\psi}{\Phi(\psi)} \tag{55}$$

one can find the bond between fluence and wave coordinate with an accuracy of arbitrary constant z_1. Thus, all the values which characterize the small velocity

wave are determined, except for the velocity itself which is not determined in the zero approximation.

In the next approximation, it is necessary to take into account in eq. (45) and other equations the linear in N_2 terms, which was not accounted for in the zero approximation, and to determine the corresponding corrections to concentrations which should depend on wave velocity. Using for the N_2 the results zero approximations (47), (52), and (53), one obtains that the corrections to the concentrations N_i, i 3 are also the functions of ψ and have first order in W. It should be mentioned that these corrections take into account another channel (except the neutron capture by ^{239}Pu) of formation of higher fuel nuclides N_i, i 3 because the nuclide N_4 is formatted as a result of intermediate nuclide burn up (this is ^{240}Pu for the uranium-plutonium cycle and ^{234}U for the thorium-uranium cycle).

As G and M are linear dependent on nuclear concentrations, the corrections to G and M have the first order in W also, and the wave stationarity conditions become

$$G^{(0)}(p,\psi) \quad WG^{(1)} \quad 0 \tag{56}$$

$$M^{(0)}(p,\psi) \quad WM^{(1)} \quad 0 \tag{57}$$

It is necessary to expend in eq. (56) all values in the power of W up to the first order terms, and take into account that the conditions of zero approximation (50) and (51) are fulfilled at $p = p_0$ and $\psi_f = \psi_{f0}$. Taking into account also the relation

$$\frac{\partial M}{\partial \psi} \quad G$$

which follows from the determination (29), one obtains that the first order corrections to p and ψ_f differ from zero. We have with accuracy to linear terms

$$p \quad p_0 \quad bW, \quad b \quad \frac{M^{(1)}}{\partial M^{(0)}/\partial p} \tag{58}$$

One can rewrite this equation in the form of wave velocity dependence on the change in the control parameter

$$u \quad \frac{L}{\tau_2 d}(p_0 \quad p) \tag{59}$$

where d is the dimensionless coefficient. The adaptability of formula (22) for wave velocity is restricted by the condition

$$W \quad \frac{u\tau_2}{L} \ll 1 \tag{60}$$

Since $d > 0$ and $W > 0$, the wave exists in the given case at p p_0 only. Namely, p_0 has the meaning of the upper limit for the absorber concentration at which the wave can exists. It should be mentioned that we have formally $p = p_0$ in zero approximation without accounting for the linear in W corrections. This can give the wrong impression that the wave exists only at $p = p_0$, and its velocity is arbitrary. As can be seen from eq. (59), this is not really the case, but in the limits of zero approximation accuracy p p_0, independently of W for the waves of small velocity. The obtained results have a general character. One can obtain additional results in specific models of nuclear kinetics and processes which have an impact on neutron balance.

THE SPECIFIC MODELS OF NFWR

The minimal (basic) model of NFWR includes such and only such elements without which the stationary wave is impossible. The construction and analysis of the minimal model is an obligatory initial step in the creation of phenomenon sequential theory, because it ensures the basic level of phenomenon qualitative understanding. Any other adequate model should contain it as a partial case and therefore is the development of the minimal model. The closer to the minimal model of NFWR is the model of Feoktistov [1]. The elements of this model which are of no consequence for the existence of NFWR are excluded from the given work and, also, the real control parameter – absorber effective concentration – is introduced instead of plutonium critical concentration, whose value is in reality variable. Together with the minimal model, we will consider below several dilated models which take into account some additional factors.

The minimal model accounts for three nuclides only: fertile (N_1), intermediate (N_2), and fuel (N_3). For the uranium cycle these are: ^{238}U, effective nuclide ^{239}Np, and ^{239}Pu, and for the thorium cycle: ^{232}Th, ^{233}Pa, and ^{233}U. An effective nuclide is a nuclide which takes into account by means of half-life change the existence of an intermediate nuclide with a small life-time (^{239}U or ^{233}Th). Nuclear kinetic equations for the stationary wave in the basic model are eqs. (44)-(46) at $\gamma_2 = 0$ (*i. e.* without effective nuclide burn up). The extensive model for the uranium cycle takes into account the kinetics of four plutonium isotopes instead of one: ^{239}Pu, ^{240}Pu, ^{241}Pu, ^{242}Pu (N_3, N_4, N_5, N_6 correspondingly) and fission products (N_7). The kinetic equation system in the extensive model, except for eqs. (44)-(46), also contains following equations

$$\dot{N}_i \quad \alpha_{i\ 1}\dot{\psi}N_{i\ 1} \quad \gamma_i\dot{\psi}N_i \tag{61}$$

$$\dot{N}_7 \quad \sum_{i\ 1}^{6}(\gamma_i \quad \alpha_i)\dot{\psi}N_i \quad \alpha_7\dot{\psi}N_7 \tag{62}$$

where $\alpha_i \quad \sigma_{ci}/\sigma_{a1}$, $\gamma_i \quad \sigma_{ai}/\sigma_{a1}$, $i \quad 4,5,6$, and $\alpha_7 = \sigma_{c7}/\sigma_{a1}$.

The neutron generation function g in diffusion eq. (1) takes the form

$$g \quad p \quad N_1 \quad \beta_3 N_3 \quad \beta_4 N_4 \quad \beta_5 N_5 \quad \beta_6 N_6 \quad \beta_7 N_7,$$
$$\beta_i \quad \frac{\nu_i \sigma_{fi} \quad \sigma_{ai}}{\sigma_{a1} \quad \nu_i \sigma_{f1}}, \quad i \quad 3 \quad 6, \qquad (63)$$
$$\beta_7 \quad \frac{\sigma_{a7}}{\sigma_{a1} \quad \nu_1 \sigma_{f1}}$$

where the concentration of the external absorber p is given constant. Using eqs. (6) and (63) we can find the form of G as a function of concentrations N_i and fluence ψ

$$G \quad p\psi \quad c_1(N_1 \quad 1) \quad \sum_{i \quad 2}^{7} c_i N_i \qquad (64)$$

where c_i are cumbersome coefficients, which are expressed through the α_i, γ_i, and β_i. Let us present those for the basic model

$$c_1 \quad \frac{\alpha_1 \beta_3}{\gamma_3} \quad 1, \quad c_2 \quad \frac{\beta_2}{\gamma_2}, \quad c_3 \quad \frac{\beta_3}{\gamma_3},$$
$$c_4 \quad c_5 \quad c_6 \quad c_7 \quad 0 \qquad (65)$$

Besides, in the initial state

$$\psi \quad 0, \; N_k \quad 0, \; k \quad 2 \quad 7, \; N_1 \quad 1 \qquad (66)$$

Let us restrict ourselves to the zero approximation in wave velocity. Then, N_2 in eq. (64). Using the approximation (52), one can subsequently solve the kinetic equations analytically

$$N_1 \quad e^{\psi} \qquad (67)$$

$$N_3 \quad \frac{\alpha_1}{\gamma_3 \quad 1}(e^{\psi} \quad e^{\gamma_3 \psi}) \qquad (68)$$

$$N_i \quad a_{1i} e^{\psi} + a_{3i} e^{\gamma_3 \psi} \quad \dots \quad a_{ii} e^{\gamma_i \psi}, \quad i \quad 4 \quad 7 \qquad (69)$$

where a_{ji} are the coefficients which are expressed through the α_k γ_k Having these solutions, one can find the fluence generation function and fluence generation momentum as the functions of fluence ψ

$$G \quad p\psi \quad b_1 e^{\psi} \quad b_3 e^{\gamma_3 \psi} \quad \dots \quad b_7 e^{\gamma_7 \psi} \quad c \qquad (70)$$

$$M \quad \frac{p\psi^2}{2} \quad b_1(1 \quad e^{\psi}) \quad b_3 \frac{1 \quad e^{\gamma_3 \psi}}{\gamma_3} \quad \dots \quad b_7 \frac{1 \quad e^{\gamma_7 \psi}}{\gamma_7} \quad c\psi \qquad (71)$$

where b_i and c are expressed through α_k γ_k, and β_k. The balance conditions $G = 0$ and $M = 0$ have the form of algebraic system for p and ψ The variable p is simply excluded, and the obtained equation for ψ is solved numerically. The obtained values of $p = p_0$ and $\psi = \psi_{f0}$ correspond to the stationary wave of nuclear burning in velocity zero approximation. Final nuclear concentrations are determined using eqs. (67) - (69) at ψ = = ψ_{f0}. The neutron flux and connection between fluence and wave coordinate are determined by eqs. (52)-(55).

Figure 1 and fig. 2 illustrate with an example of the basic model the work of balance conditions (32) and (33) or, more exactly, the conditions of zero approximation (50) and (51). The phase trajectories of the system in plane (ψ, $\dot{\psi}$), calculated with eqs. (50) and (51) for three different values of absorber concentration p are presented in fig. 1. Trajectories 1 and 3 for p p_0 and p p_0, correspondingly, have no physical meaning, since for the first one $\dot{\psi}$ changes the sign (part of the curve with $\dot{\psi} < 0$ is not shown), and for the second one $\dot{\psi}$ is infinitely increased. In reality, only trajectory 2 can exists, and it corresponds to the stationary wave at $p = p_0$. This trajectory begins at the point ψ 0, $\dot{\psi}$ 0 (initial state) and finishes at the point ψ ψ_{f0}, $\dot{\psi}$ 0 (final state). Besides, the wave coordinate changes from –∞ to +∞.

The dependence of the fluence generation function G on ψ at $p = p_0$, is presented in fig. 2, which corresponds to the curve 2 in fig. 1. At $p = p_0$ and $\psi = \psi_{f0}$ both balance conditions are fulfilled simultaneously: $G = 0$ and $M = 0$.

The parameters of the stationary wave p_0, and ψ_{f0}, calculated in zero approximation, and final values of nuclear concentrations for the basic model and several variants of the extensive model are presented in tab. 1. This illustrates the influence of different elements of the model on wave characteristics in NFWR.

The first and second lines of tab. 1 refer to the basic model. Changing the parameters of nuclide N_3, we try to take into account (to some degree) not only ^{239}Pu, but all other plutonium isotopes. In the first line, the cross-sections for N_3 correspond to ^{239}Pu. Other plutonium isotopes are not accounted for. For the second line, we take that absorption and fission cross-sections for N_3 are the same and equal to the fission cross-section of ^{239}Pu. We take into account that other plutonium isotopes can also be fissile. In reality, we take that capture and fission cross-sections and average neutron number per fission for all plutonium isotopes are the same as for ^{239}Pu. In such a case, the concentration of N_3 corresponds to the sum of all plutonium isotope concentrations.

It is obvious that such a change of N_3 parameters leads to an increase in fission neutron number and, in order to compensate this, absorber concentration should be increased: as can be seen from tab. 1, p_0 is increased sufficiently. But, the final fluence ψ_{f0} decreases, and according to eqs. (67) and (68), final concentrations N_1 and N_3 increase.

The third line differs from previous ones by taking into account all four plutonium isotopes instead of single N_3, but it does not account for the absorption by fission products as in the basic model. This gives the intermediate value of p_0 between the first two lines. But, as is seen, the final fluence ψ_{f0} is maximal compared to the previous two. The final concentration of

Table 1. The parameters of the stationary wave for different NFWR models

Model description		p_0	ψ_m	Final values of the dimensionless concentrations					
Isotopes of Pu	Absorption by fission products			N_1	N_3	N_4	N_5	N_6	N_{Fp}
^{239}Pu, σ_{e3} 0	no	0.335	1.770	0.171	0.021	–	–	–	–
^{239}Pu, σ_{e3} 0	no	0.711	1.604	0.201	0.032	–	–	–	–
^{239}Pu, ^{240}Pu ^{241}Pu, ^{242}Pu	no	0.448	1.935	0.144	0.018	0.019	0.004	0.003	–
^{239}Pu, ^{240}Pu ^{241}Pu, ^{242}Pu	yes	0.083	1.123	0.325	0.040	0.035	0.006	0.003	0.284
^{239}Pu, ^{240}Pu ^{241}Pu	yes	0.082	1.121	0.326	0.040	0.034	0.006	–	0.283

fertile nuclide N_1, accordingly, decreases. This points to the two-parameter character of the system: the stationary wave of small velocity is characterized by two parameters p_0 and ψ_{f0}, which are changed independently with changes in the contents of the model and its parameters.

The forth line differs from the third one by accounting for fission products, their accumulation, burning and neutron absorption by them. It can be seen, first of all, that all parameters of the wave have cardinally numerically changed compared to line 3. The appearance of an additional neutron absorption channel leads to a sharp decrease in p_0: the change of neutron absorption by absorber to absorption by fission products occurs within the general neutron balance.

But, it can also be seen that such a change of one absorption mechanism to another is not equivalent: the final fluence is approximately half, and accordingly, burn up decreases and final nuclide concentrations change. This is connected with the change of neutron generation momentum. It is sufficient that the absorber and fission product insertions into the neutron generation function are distributed in a different way along the fluence coordinate. The first one is constant, and the second one increases together with fission products accumulation (fig. 3).

The fifth line shows that the isotope ^{242}Pu practically does not have an influence on wave parameters.

Figures 3 and 4 show the dependency of fluence on nuclear concentrations with the final fluence value corresponding to the line 4 of tab. 1. One can get the real wave profile by some non-linear dilatation of these curves along the abscissa axis according to formula (55), as a result of which, the finite interval (0, ψ_{f0}) turns into an infinite interval ($-\infty$, ∞) of wave coordinate or x, or t.

Let us emphasize that the dependencies (67)-(69) are determined by the nuclear kinetic equations only, and therefore, are the same for all variants of tab. 1, excluding line 2, for which the cross sections of N_3 were changed. The difference is in different final values of fluence $\psi = \psi_{f0}$ of nuclear fission termination. Therefore, the final nuclear concentrations and wave profiles along the coordinates x and t are different also.

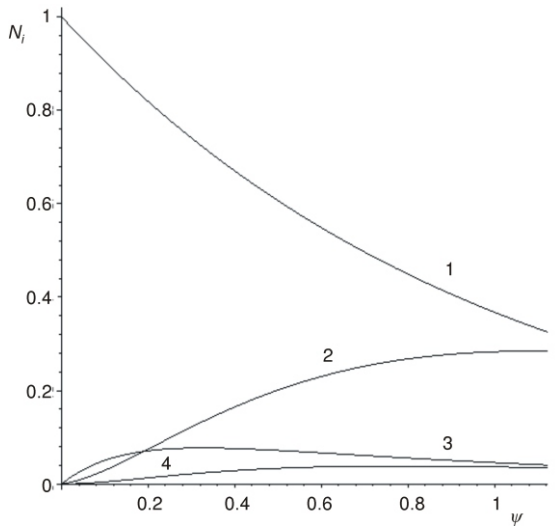

Figure 3. The dependencies of the dimensionless concentrations on the dimensionless fluence
1, 2, 3, and 4 denote concentrations N_1, N_7, N_3, and N_4 correspondingly

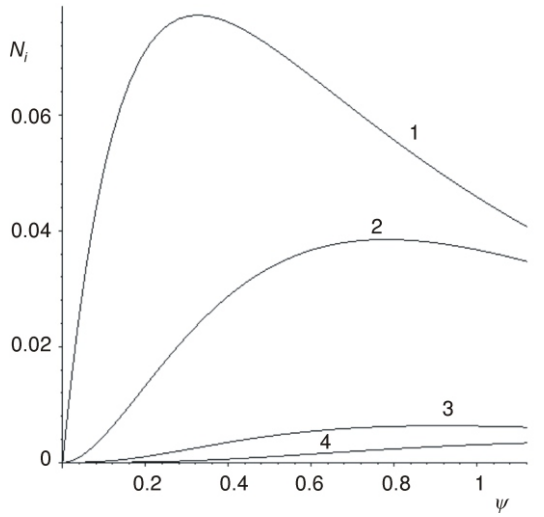

Figure 4. The dependencies of the dimensionless concentrations on the dimensionless fluence
1, 2, 3, and 4 denote accordingly ^{239}Pu, ^{240}Pu, ^{241}Pu, and ^{242}Pu

The small corrections in W to the nuclear and neutron concentration to p and ψ_f appear in the next linear approximation of wave velocity. Due to them, the wave velocity has a definite value at the given absorber concentration $p < p_0$. The same compensatory mechanism appears with changing p, which due to the change of wave velocity and ψ_f eliminates the imbalance in neutron generation and momentum generation function. Besides, the necessary change in velocity is determined by the change of the momentum generation function with changing p.

This compensatory mechanism appears due to two first orders in W terms in eq. (45) for the concentration of the effective nuclide N_2: these are term on the left hand side and the last term on the right hand side. Actually, two different parallel mechanisms are at work here. The first one is connected with the accumulation and subsequent consumption of nuclide N_2, which leads to a delay in the creation of N_3 relative to a decrease in N_1 concentration. In velocity zero approximation such a delay is absent. The second mechanism is connected with the burning of nuclide N_2, and correspondingly, with the creation of nuclide N_4 (^{240}Pu) instead of nuclide N_3 (^{239}Pu). Both effects result in corrections to the generation function.

The deposits of both mechanisms are comparable. They have one order in wave velocity and are principal terms in wave velocity determination. Therefore, the velocity of a nuclear fission wave in general can not be calculated adequately in every complex model, if the said model does not take into account the "small" effects of effective nuclide burning.

CONCLUSIONS

In spite of an obvious correspondence of contents and nuclear processes at the microscopic level, the usual fast reactor and NFWR are fundamentally different in many features at the system level.

The NFWR is an autonomic system and the usual fast reactor is not autonomic (except for short intervals of control system non-intervention). Due to autonomy, fluence ψ in a NFWR can play the role of the internal variable of the system determining its current state and space – time evolution of the system during the propagation of the stationary wave of small velocity as a unidirectional movement along the fluence coordinate from the initial to the maximal final value. Current values of nuclear concentrations and neutron flux are determined by the current value of fluence ψ only.

The value of criticality itself is not sufficient for the analysis of steady-state wave regime in NWFR. The value which really determines the internal equilibrium in the nuclear fission wave is neutron generation function's dependence on fluence.

In point kinetics approximations for usual reactors, the spatial concentration distribution is fixed, only their amplitude changes and reactor dynamics are time dynamics. Contrary to this, in NFWR, the stationary wave is formatted by two concordant processes: spatial quasistationary neutron diffusion and nuclear concentration time kinetics. As a result, instead of one criticality condition required by the usual reactor, the stationary wave regime in NFWR requires two balance equilibrium conditions. These conditions are analogous to the conditions of solid mechanical equilibrium in the field of external forces: the zero and first momentum of neutron generation function integrated over fluence should be equal to zero. Practically, this means that not only the full number of generated and absorbed neutrons during wave propagation should be balanced, but that their distributions inside the wave should also be balanced, namely that their centers of area along the fluence coordinate should coincide.

As can be seen from presented calculations, the dimensionless fluence in NFWR reaches the values ψ – , and the level of fuel burn up (fertile nuclide) is >30%, which is significantly higher than in usual reactors. Therefore, the processes, which are usually not of great importance in fast reactors, can play a significant role in NFWR at large levels of burn up. Accordingly, traditional approximations for fast reactors are not congenial to NFWR and should be revised. This is, for example, relevant when fission products are concerned, since the results for different NFWR models show that neutron absorption by fission products has sufficient numerical influence on wave parameters.

External absorber addition to fuel content gives us the possibility of controlling wave velocity and lowering its value to an acceptable level of energy release from a practical point of view. In the actual range of small wave velocities, velocity decreases linearly to absorber concentration growth, while a stationary wave does not exist when the absorber limit concentration, which formally corresponds to zero wave velocity, is exceeded. The necessities for the existence of a stationary wave of small velocity absorber concentration and final fluence are mainly determined by the cross-sections of nuclides contained in the reactor, and are dependent on wave velocity to a very small extent.

REFERENCES

[1] Feoktistov, L. P., Neutron-Induced Fission Wave, *Soviet Physics Doklady*, *34* (1989), pp. 1071-1073, Doklady Akademii Nauk SSSR 309 (1989), pp. 864-867 (in Russian)
Feoktistov, L. P., Safety: The Key to Revitalization of Nuclear Power, *Phys.-Usp., 36* (1993), 8, pp. 733-743

[2] Teller, E., Nuclear Energy for the Third Millennium, – Preprint, UCRL-JC-129547, LLNL, 1997

[3] Goldin, V. Ya., Anistratov, D. Yu., Fast Neutron Reactor in the Self Regulating Neutron – Nuclear Regime (in Russian), *Math. Modeling, 7* (1995), 10, pp. 12-32
[4] Goldin, V. Ya., Pestriakova, G. A., Troshchiev, Yu. V., Aristova, E. N., Fast Reactor on Oxide Uranium– Plutonium Fuel in the Self Regulating Regime (in Russian), *Atomic Energy, 94* (2003), 3, pp. 184-190
[5] Sekimoto, H., Ryu, K., Yoshimura, Y., A New Burnup Strategy CANDLE, *Nucl. Sci. Eng., 139* (2001), 3, pp. 306-317
[6] Sekimoto, H., Tanaka, K. Application of CANDLE Burnup Strategy to Small Reactors, *Trans. Am. Nucl. Soc., 87* (2002), 4, pp. 399-400
[7] Sekimoto, H., Ohoka, Y., Application of CANDLE Burnup to Block-Fuel High Temperature Gas Reactor, *Proceedings on CD,* ICAAP'03, Cordoba, Spain, May 4-7, 2003, . Paper 3013
[8] Fomin, S., Mel'nik, Yu., Pilipenko, V., Shul'ga, N., Investigation of Self-Organization of the Non-Linear Nuclear Burning Regime in Fast Neutron Reactors, *Annals of Nuclear Energy, 32* (2005), 12, pp. 1435-1456
[9] Fomin, S. P., Mel'nik, Yu. P., Pilipenko, V. V., Shul'ga, N. F., Study of Self-Organaizing Regime of Nuclear Burning Wave in Fast Reactor, *Problems of Atomic Science and Technology, Series: Nuclear Physics Invest*., 6 (2005), 45, pp. 106-113
[10] Rusov, V. D., Pavlovich, V. N., Vaschenko, V. N., Tarasov, V. A., Zelentsova, T. N., Bolshakov, V. N., Litvinov, D. A., Kosenko, S. I., Byegunova, O. A., Geoantineutrino Spectrum and Slow Nuclear Burning on the Boundary of the Liquid and Solid Phases of the Earth's Core, *Journal of Geophysical Research*, *112* (2007), B09203, DOI:10.1029/2005JB004212
[11] Van Dam, H., Self-Stabilizing Criticality Waves, *Annals of Nuclear Energy, 27* (2000), 12, pp. 1505-1521
[12] Van Dam, H., Flux Distributions in Stable Criticality Waves, *Annals of Nuclear Energy, 30* (2003), 12, pp. 1495-1504
[13] Chen, X.-N., Maschek, W., Transverse Buckling Effects on Solitary Burn-up Waves, *Annals of Nuclear Energy, 32* (2005), 12, pp. 1377-1390
[14] Pavlovich, V. N., Khotyaintseva, E. N., Rusov, V. D., Khotyaintsev, V. N., Yurchenko, A. S., Reactor Operating on a Slow Wave of Nuclear Fission, *Atomic Energy, 102* (2007), 3, pp. 151-158

Владимир М. ПАВЛОВИЧ, Владимир М. ХОТЈАЈИНЦЕВ, Олена М. ХОТЈАЈИНЦЕВА

ФИЗИЧКА ОСНОВА ФИСИОНОГ ТАЛАСНОГ РЕАКТОРА

Разматра се идеја водиља фисионог таласног реактора на споре неутроне и приказује кратак преглед постојећих радова. Сврха је да се расветли физика процеса који оцртавају стационарни талас нуклеарног изгарања и да се развију поступци за одређивање таласних параметара. Показано је да се код таласа нуклеарне фисије дифузиона једначина по флуенсу може користити за опис стационарних и нестационарних процеса. У раду су најпре утврђена два услова за егзистенцију стационарног таласа, а потом је формулисано правило за одређивање брзине таласа као својствене вредности граничног проблема.

Кључне речи: реактор, фисија, аутоталас, нуклеарно изгарање