FRACTIONAL VARIATIONAL PROBLEMS AND PARTICLE IN CELL GYROKINETIC SIMULATIONS WITH FUZZY LOGIC APPROACH FOR TOKAMAKS

by

Danilo RASTOVIĆ

Received on April 15, 2009; accepted in revised form on August 6, 2009

In earlier Rastovic's papers [1] and [2], the effort was given to analyse the stochastic control of tokamaks. In this paper, the deterministic control of tokamak turbulence is investigated via fractional variational calculus, particle in cell simulations, and fuzzy logic methods. Fractional integrals can be considered as approximations of integrals on fractals. The turbulent media could be of the fractal structure and the corresponding equations should be changed to include the fractal features of the media.

Key words: fractional variational calculus, particle in cell simulations, fuzzy logic, Hamiltonian, tokamak

INTRODUCTION

Functional minimization problems naturally occur in engineering and science where minimization of functionals, such as Lagrangian, potential, total energy, *etc*., gives the laws governing the system behaviour. In the optimal control theory, minimization of certain functionals gives control functions for the optimum performance of the system.

Recent investigations have shown that many physical systems can be represented more accurately using fractional derivative formulation. A fractional calculus of variation problem is a problem in which either the objective functional or the constraint equations or both contain at least one fractional derivative term [3].

We can consider the problem of Lagrange containing fractional derivatives, or simply a fractional Lagrange problem. We obtain the Euler-Lagrange equations for unconstrained and constrained variational problems via the fuzzy logic approach.

Nuclear Technology & Radiation Protection
Technical paper
UDC: 621.039.52:517.97
DOI: 10.2298/NTRP0902138R

Author's address:
Control Systems Group, 62, Nehajska Str.
10110 Zagreb, Croatia

Author's e-mail:
dr.rastovic@excite.com

A FRACTIONAL VARIATIONAL CALCULUS

A symplectic structure on a manifold is a closed non-degenerate differential 2-form. The phase space of a mechanical system has a natural symplectic structure. On a symplectic manifold, as on a Riemannian manifold, there is a natural isomorphism between vector fields and 1-forms. A vector field on a symplectic manifold corresponding to the differential of a function is called a Hamiltonian vector field. The phase flow of a Hamiltonian vector field on a symplectic manifold preserves the symplectic structure of phase space.

The following example explains the appearance of symplectic manifolds in dynamics. Along with the tangent bundle of a differential manifold, it is often useful to look at its dual – the cotangent bundle. The cotangent bundle has a natural symplectic structure. A differential k-form is called an integral invariant of the map g if the integrals of k-form on any k-chain c and on its image under g are the same.

The function H is the first integral of the Hamiltonian phase flow with the Hamiltonian function H. We obtain yet another generalization of E. Noether's theorem: given a flow which commutes with the one under consideration, one can construct the first integral. The technique of generating functions for canonical transformations, developed by Hamilton and Jacobi, is the most powerful method available for integrating the differential equations of dynamics.

All the basic propositions of Hamiltonian mechanics follow directly from Stokes' lemma. For example, we obtain Liuville's theorem: the phase flow preserves area. The mapping g is called canonical if g preserves the 2-form. Canonical transformations preserve the volume element in the phase space. Suppose now that the Hamiltonian function $H(p, q)$ does not depend on time. Then the canonical equations have the first integral: $H[p(t),q(t)]$ = const.

The universal principle described by Noether's theorem asserts that invariance of the integral functionals of the calculus of variations with respect to a family of the transformation result in existence of certain conservation law or equivalently the first integral of the corresponding Euler-Lagrange differential equations. This means that the invariance hypothesis leads to quantities which are constant along the extremals. A quadruple satisfying the Hamiltonian system and the maximality condition is called a Pontryagin extremal.

The possibility of invariance up to first-order terms in the parameter (quasi-invariance) is crucial. The group concept is not required for the investigation of quasi-invariant optimal control problems [4]. The notion of conservation law – the first integral of the Euler-Lagrange equations – is well known in physics.

One of the most important conservation laws is the integral of energy. In classic mechanics, beside the conservation of energy, the conservation of momentum or angular momentum may occur. These conservation laws are very important: they can be used to reduce the order of Euler-Lagrange differential equations, thus simplifying the resolution of the problem.

A formulation of the Euler-Lagrange equations was given for problems of the calculus of variations with fractional derivatives. A Noether's theorem for the fractional Euler-Lagrange extremals is proven [5]. There is no general formulation of a fractional version of Pontryagin's Maximum Principle. With a fractional notion of Pontryagin extremal, one can try to extend it to the more general context of the fractional optimal control.

PHYSICS OF TOKAMAK

Fluctuation phenomena are important to many physical systems, such as magnetically confined plasmas. The relaxation of the fluctuations is modeled by the diffusion equation. Earlier, an approach for an anomalous diffusion with the Rebut's concept was written [6]. The spatial correlations of random fluctuations are modeled by the exponential decay. Based on these models, the temporal correlations of random fluctuations, such as the correlation function and the power spectrum, are calculated [7].

In the turbulent state, the system is irregular in both space and time. Therefore, we need to study both spatial and temporal correlations of random fluctuations. During a steady-state plasma discharge, the fluctuations in the magnetized plasma usually have the following properties: the time series of the signals are nearly random, the statistics of the fluctuations are rougly homogenous and isotropic and the correlation length is shorter than the length scales of the background density and temperature gradients.

Periodic orbits are important objects in the study of dynamical systems. In conservative systems, the winding number of periodic orbits in the integrable regime indicates the transitions to chaos as well as topological structure of the phase space. In dissipative systems, the structural properties of strange attractors such as dimension, Liapunov exponents, and topological entropy can be determined using unstable periodic orbits. Even the existence of determinism in experimental time series can be revealed by the presence of periodicity in the experimental data. In the variational method, one defines a certain cost function and a periodic orbit is obtained by seeking a curve that minimizes the cost function [8]. In the conventional approach of using a Poincaré surface of section, close returns need to do two things: intersect the Poincaré surface and be close to the starting point.

There are numerous examples of rare intense events in spatiotemporal chaotic systems.

Tokamaks are highly sensitive to magnetic perturbations that break toroidal symmetry. Such pertubations always exist due to the imperfections in magnetic field coils or the presence of magnetic materials. By finding the perturbed equilibrium given by each Fourier component, one can obtain the total resonant field at each rational surface. The localization of the dominant mode, or the first mode, corresponds to a broad poloidal harmonic coupling in toroidal plasmas.

The optimization of the current profile means the control of the current profile. In a long-pulse operation aiming at the steady-state operation for the fusion reactor, the preprogrammed control, or external current drivers, is not enough for the current profile control, since the plasma must undergo various stages having different confinement and bootstrap current profile, such as current ramp-up, H-mode transition and internal-transport-barrier (ITB) formation. Thus, the current profile control in real-time during the discharge is required.

Perturbative experiments in magnetically confined fusion plasmas have shown that edge cold pulses travel to the centre of the device on a time scale much faster than expected on the basis of diffusive transport. An open issue is whether the observed fast pulse propagation is due to non-local transport mechanisms or if it could be explained on the basis of local transport models [9]. To elucidate this distinction, perturbative experiments involving ion-cyclotron resonance heating (ICRH) power modulation in addition to cold pulses have been conducted in Joint European Torus (JET) for the same plasma.

What does it mean if a system is Hamiltonian? Is it useful to know? Every system can be presented as a subsystem or a factor-system of the Hamiltonian one. The purpose is to present recent advances in theory without the uniform stability condition.

Plasma equilibrium codes generally solve numerically Grad-Shafranov and Maxwell equations on a tokamak two-dimensional (2-D) finite element meshing assuming a global axial symmetry to compute the poloidal flux and magnetic field everywhere in a meridian plane of the machine.

Assuming an symmetric plasma in a cylindrical coordinate system, the equilibrium MHD equations (fluid mechanics equations + Maxwell's equation) is reduced to the Grad-Shafranov equation which describes the force balance of the tokamak equilibrium.

The efficient and safe operation of fusion devices relies on the accurate knowledge of many of the discharge parameters. The values of several discharge parameters which are not directly measurable, such as plasma shape and current density distribution, can be reconstructed from the magnetic field and flux measurement.

A variety of the poloidal field coil systems could support the specified range of plasma equilibria. It is necessary to determine the optimal choice. The optimization requires the generation and analysis of a large number of magnetohydrodynamic (MHD) equilibria. A few thousand equilibria may have to be computed.

The macroscopic equilibrium is the result of the complex dynamics of the underlying system of particles subject to individual as well as the collective interactions. One could ask whether these interactions may result finally in a spontaneous selection among the infinite possibilities, a selection operated innerly by plasma itself when it is not forced artificially towards a specific configuration by external interventions [10]. One is led naturally to consider that suitably defined probabilities, whose relative measure determines the selection among all possible states of the fluctuating system, could correspond to the equilibrium states of the plasma. We have adopted the point of view of the information theory where the constraints have a hypothetical meaning to be confirmed by the experimental test.

Functional of the density of current is given. Related vector potentials, whose properties of variations determine the privileged plasma equilibriums in the so-called thermodynamic limit, are introduced. The tokamak is an open system in interaction with the auxiliary power. The entropy is not required to be at maximum value, but it can be assumed to be stationary, even locally, expressing the local balance between the entropy injected externally, and the entropy produced in plasma.

The natural field of application of the stationary magnetic entropy model concerns relaxed states in which the dissipation processes are counterbalanced by external sources (ohmic or auxiliary) such that the magnetic entropy and the plasma-state are constant in time, at least approximately.

We have problems with the particle, energy, and momentum transport in shaped plasmas. The shape of the density profile is expected to have a significant impact on the performance of the International Thermonuclear Experimental Reactor (ITER) and fusion reactors. Peaked profiles can increase the fusion power, but may also lead to a deleterious impurity accumulation in the reactor core. The behaviour of particle and impurity density profiles is investigated in a number of different scenarious, including electron cyclotron heating (ECH) and electron cyclotron current drive (ECCD) low (L)-modes, electron internal transport barriers (eITBs) and high (H)-modes, with a view to improving our physics understanding and predictive capability for electron heated, ignited reactor conditions.

Strictly speaking, a system of charged dust particles cannot be described by a Hamiltonian, since the energy is not conserved because of the openess of the system due to plasma – particle interaction. Under some conditions, however (*e. g.* when the energy flow in and out of the system is balanced), the Hamiltonian treatment provides useful insights. A potential can still be defined in tokamaks so that for a certain set of parameters the system is described by the Hamiltonian. When the system loses stability, the traditional Hamiltonian analysis fails and we must explicitly analyse the dynamical equations of motion. Since most systems in nature are non-Hamiltonian, it can be useful to redefine the Hamiltonian to include non-potential forces. The class of non-Hamiltonian systems can be described by a non-holonomic (non-integrable) constraint: the velocity of the elementary phase volume change is directly proportional to the power of non-potential forces. The coefficient of this proportionality is determined by Hamiltonian.

PARTICLE-IN-CELL SIMULATIONS

Five-dimensional (5-D) gyrokinetic simulations are essential tools to study anomalous turbulent transport in tokamak plasmas. Global toroidal gyrokinetic simulations have been developed either by particle (Lagrangian) approaches or by mesh (Eulerian or semi-Lagrangian) approaches. Most of particle simulations are based on a particle-in-cell (PIC) method which is valid for an isolated system without any sources and collisions.

In developing global gyrokynetic full-f Vlasov simulations, it is extremely important to use the modern gyrokinetic theory which keeps the Hamiltonian structure of the problem [11]. Since the modern gyrokinetic theory is based on the Hamiltonian or Lagrangian formalism, the equation system keeps the Hamiltonian structure and the corresponding first principles. These first principles are important for the verification of simulation codes.

An exact equilibrium solution of the collisionless gyrokinetic equation satisfying an equilibrium condition is given by a so-called Vlasov equilibrium or an arbitrary function of constants of motion in the unperturbed characteristics. Collisionless gyrokinetic PIC simulations of the ion temperature gradient (ITG) driven turbulence with a Vlasov equilibrium and local Maxwellian distribution are clarified.

The first direct implicit ion polarization gyrokinetic full-f particle-in-cell code has been written and is implemented with kinetic electrons in global tokamak transport simulations. The code is applicable for calculations of rapid transients and steep gradients in plasma and is thus amenable, *e. g.* for studies of transport barrier formation.

A gyrokinetic particle-in-cell approach is applicable for calculation of the evolution of particle distribution function f including, as special cases, strong plasma pressure profile evolution by transport and formation of neoclassical flows. The code has been validated against the linear predictions of the unstable ion temperature gradient mode growth rates and frequencies. Convergence and saturation in both turbulent and neoclassical limit of the ion heat conductivity is obtained by numerical noise well suppressed by a sufficiently large number of simulation particles [12].

Large-scale kinetic simulations of toroidal plasma dynamics based on the first principle are called for in investigations on such transient transport mechanism like low (L) to high (H) confinement transport barrier formation or edge localized modes (ELM) at the edge plasma, internal transport barrier (ITB) formation in the core plasma, or intermittend turbulence in magnetic fusion devices.

After initialising ions on collisionless orbits, one electron is initialized for each ion, at the same location to ensure quasi-neutrality. In more realistic simulations, pairwise reinitialization of outflowing ions and electrons according to assumed neutral distribution or ionization on randomly directed straight return paths of recycled neutrals is chosen. For steeper density gradient and higher temperature, the physical density fluctuations are much better resolved at saturation according to the mixing-lenght prediction.

A relative large variety of gyrokinetic and gyrofluid simulation models for investigating microinstabilites and turbulence exists, and to build confidence in the result produced by these models, a rigorous benchmarking procedure for cross-validation must be adopted. Traditionally, in delta f codes the linear analysis is performed using a local Maxwellian background distribution, which is simple to implement and produces adequate results. Particle-in-cell simulations were used for the calibration of the probe for ion and electron temperature measurement [13].

A view of the mechanisms underlying particle and energy transport in fusion plasmas devices (tokamak and stellarators) is given. The heat and energy transport in fusion plasmas is generally due to the turbulent processes. Several driving (plasma gradients) and damping (electric field) mechanisms of turbulence have been investigated. A new approach based on multi-field probability density function statistical analysis has been proposed to describe transport in non-equilibrium systems and to unravel the overall picture connecting transport, gradients and flows in fusion plasmas [14].

Kolmogorov's theory and philosophy of turbulence are based on a number of assumptions which have become standard notions with the help of which one approaches turbulence in many, including non-hydrodinamic, systems. However, it turns out that in MHD turbulence, locality of interactions in scale space, isotropy of small scales or even universality cannot be taken for granted, and, in fact, can be shown to fall [15]. Tokamak disruptions give large-scale explosive events. Explosive events almost always require non-linear destabilization to achieve the fast-time scales that are observed. The system crosses the instability threshold in a small region of space.

FUZZY LOGIC DESIGN

The conceptual tools of fuzzy logic are fully transparent, since their governing rules are human defined. They can therefore help not only in forecasting the disruptions of tokamaks but also in studying their behaviour [16]. Since tokamak fusion plasmas are complex, non-linear systems, driven far from equilibrium by powerful additional heating systems, many causes can destabilize them and cause a disruption. Therefore, due to the lack of algorithmic solutions, many efforts were devoted to predicting disruptions using neural networks in the past. An alternative approach to disruption prediction for JET is presented, based again on fuzzy logic but with an adaptive character.

In some plasma configurations, usually called advanced scenarios, an ITB can appear. These barriers considerably reduce the transport of particles and energy from the inner to the outher side of plasma. An ITB requires a steep pressure gradient that can lead to instabilities and disruptions. A fuzzy inference system was previously devised, to predict the proximity of a disruption at JET and to compare its performance with other methods. This predictor is called static because it was trained to work for whole discharges, without any allowance for different phases during the shot.

The correlation analysis of the inputs proves the limitations of a static approach. At different levels of disruption probability, the same diagnostic signal can carry information of significantly different importance and therefore must be weighted accordingly. Consequently, a prototypical adaptive predictor has been developed, which consists of a set of different fuzzy systems optimized for different disruption probabilities.

The autonomous Lagrangian, corresponding to a mechanical system of conservative points, is invariant under time-translations (time – homogeneity symmetry), and from Noether's theorem follows that the total energy of a conservative closed system always remains constant in time, that it cannot be created or destroyed, but only transferred from one form into another.

The main result in [17] gives that the total energy (the autonomous Hamiltonian) of a fractional system is not conserved. The Euler-Lagrange equations for fractional optimal control problems are obtained, using the traditional approach of the Lagrange multiplier rule. The Hamiltonian defines a conservation law only in the integer case $n = 1$.

The Liouville and first Bogoliubov hierarchy equations with derivatives of non-integer order are derived [18]. The fractional Liouville equation is obtained from the conservation of probability to find a system in a fractional volume element. The linear fractional kinetic equation for distribution of the charged particles is considered. Typical turbulent media could be of a fractal structure, and the corresponding equations should be changed to include the fractal features of the media. The equilibrium for fractal turbulent media can exist if the magnetic field for magnetohydrodynamics equations satisfies the power law relation. We can derive the fractional generalization of Gauss' theorem and also the fractional generalization of Stokes' theorem.

In the paper [19], the approximation of the surface and volume integrals of the unknown functions, which is in a particle as sums of these function values in the points (particles) which are in the nearest vicinity to the given one, is presented. The volume integral is equal to the sum $A(i, j) f(j)$, the surface integral is equal to the sum $B(i,j) f(i,j)$ over the set of numbers of neighbor particles for the particle number i, $f(i, j)$-approximation of function f at the „fuzzy face" between i-th and j-th particles. The coefficients $A(i, j)$, $B(i, j)$ should satisfy some relations. So, for each particle we have something like a cloud of particles around it, which is used for the approximation construction. The term particle is used as close as can be to the reality one, but really we have a volume, which doesn't have strict bounds, but about which we know values, which is necessary for modeling of a specific applied problem. The described idea makes it easy to implement any finite volume difference scheme in its frame. The scheme can be easily formulated in finite volume terms, thus making natural its 2-D and 3-D formulation for fuzzy grid. This method can be easily extended to the case of fractional volume elements (*i. e.* fractional Gauss's theorem and the fractional Stokes' theorem) and we obtain fractional fuzzy grid.

An adaptive fuzzy control can be applied to synchronize such kinds of dynamical behaviour. The proposed method must be robust to approximate errors and disturbances, because it integrates the merits of adaptive fuzzy and variable structure control. The similar method for another problem is described in [20]. According to the behaviour of our target probability distribution functions, the fuzzy actuators should be derived.

The advantage of the recurrence based method in comparison to other standard techniques is that it is possible to distinguish between quasiperiodic and chaotic orbits that are temporarily trapped in a sticky domain, from very short trajectories. The difference between the recurrence properties and quasiperiodic and chaotic orbits helps to understand the complex patterns of the corresponding recurrence plots. Several measures from the recurrence quantification analysis can be used to quantify these patterns.

The fuzzy control algorithm is used to stabilize the fixed points of a chaotic system [21]. No knowledge of the dynamic equation of the system is needed in this approach and the whole system is considered as a black box. It can be used for investigation of the behaviour of the fractional Vlasov-Poisson-Fokker-Planck system.

In the paper [22] the sawtooth crash in the axially symmetric divertor experiment (ASDEX) upgrade tokamak is investigated and evidence supporting the hypothesis of stochastization of magnetic field lines during the crash is presented. On the basis of soft X-ray and electron cyclotron emission measurements that occur during the pre-crash phase the quasiperiodic transition to chaos is investigated. In the paper [23] some preliminary results concerning the use of artificial neural networks (ANN) for the feedback stabilization of a thermonuclear reactor at nearly ignited burn conditions are presented. An algorithm to train ANN for this purpose, involving the method of backpropagation through time and the conjugate gradients technique, is discussed. Disruption is a sudden loss of magnetic confinement that can cause a damage of the machine walls and support structures. For this reason, it is of practical interest to be able to early detect the onset of the events with vizualization, maximum entropy and complexity as corrections methods.

CONCLUSIONS

Using the well-known Ostrogradski or Gauss formula, the existence and uniqueness of the theorems for the classical weak solution of the model in some cases can be proven. Fuzzy logic, genetic algorithms and artificial neural networks are not competing each other, but instead they may be combined on the basis of integrated frameworks to outperform conventional design approaches [24]. The genetic algorithms are numbers of collections of methodologies known as evolutionary computation (EC).

During the last decades, researches have been using fuzzy logic as a representation framework in design problems characterized by inherent uncertainty during decision-making. The transition from the Aristotelian logic (between two competing states one and only one is true) to the fuzzy logic (multiple competing states may be true at the same time, each one at the different degree of truth) was hesitant. The ability, however, of modeling the uncertainty through fuzzy logic attracted many researches that contributed to the foundation of various fuzzy logic concepts relative to the engineering design.

If a fuzzy logic based technique is utilized in solving a problem, then the designer should know exactly the impact of using the alternative methods for the aggregation and defuzzification of the fuzzy components. The sustainability and the success of an advanced technique should always be based on simplicity and this simplicity can originate in the combination of characteristics of the three design models: prescriptive, descriptive, and computer-based.

Various experimental issues in the exploration require drastic improvements of plasma real-time control functions based on the understanding of a plasma equilibrium state [25]. In particular, precise reproduction methods of plasma shape and current profile are the key elements for any advanced control. Plasma shape in a tokamak often determines some of the basic parameters for energy confinement, stability and safe operation. The reproduced position and shape are directly utilized for their feedback control. In addition, the full plasma shape is visualized on the screen as a real-time monitor.

We obtained the possibility of simulations of tokamak plasma behaviour for Hamiltonian and non-Hamiltonian flow via fuzzy fractional variational calculus description. In the case of non-existence theorem of plasma behavior for such kind of variational calculus, we can use the methods of fuzzy simulations that are explained in the papers [26-30].

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Данило РАСТОВИЋ

ФРАКТАЛНИ ВАРИЈАЦИОНИ ПРОБЛЕМИ И ЖИРОКИНЕТИЧКЕ СИМУЛАЦИЈЕ ЧЕСТИЦЕ У ФАЗНОЈ ЋЕЛИЈИ СА ФАЗИЛОГИЧКИМ ПРИСТУПОМ ЗА ТОКАМАКЕ

Познато је да се понашање плазме код токамака описује помоћу Колмогоров-Арнолд-Мосерове теореме, те се фазни простор Хамилтонијана дели на стохастички и детерминистички део. У ранијим радовима аутора [1, 2], у циљу добијања рекурентног понашања (поопштења периодичности) примењене су методе вештачке интелигенције за стохастички случај, тј. за фрактално Брауново кретање. У овом чланку, на основу приказа резултата аналитичке механике, објашњава се нужност увођења метода вештачке интелигенције за фрактални детерминистички случај добијања рекурентног понашања преко Канторуса за контролу процеса у реалном времену. Преостаје да се синхронизује излаз детерминистичког и стохастичког случаја, у сврху добијања рекурентног понашања токамака дугог пулсирања у стању баланса равнотеже.

Кључне речи: фрактални варијациони рачун, симулације честице у фазној ћелији, фази логика, Хамилтонијан, токамак