

EVALUATION OF RADIATION HEAT TRANSFER IN POROUS MEDIA

Application for a pebble bed modular reactor cooled by CO₂ gas

by

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This work analyses the contribution of radiation heat transfer in the cooling of a pebble bed modular reactor. The mathematical model, developed for a porous medium, is based on a set of equations applied to an annular geometry. Previous major works dealing with the subject have considered the forced convection mode and often did not take into account the radiation heat transfer. In this work, only free convection and radiation heat transfer are considered. This can occur during the removal of residual heat after shutdown or during an emergency situation. In order to derive the governing equations of radiation heat transfer, a steady-state in an isotropic and emissive porous medium (CO₂) is considered. The obtained system of equations is written in a dimensionless form and then solved. In order to evaluate the effect of radiation heat transfer on the total heat removed, an analytical method for solving the system of equations is used. The results allow quantifying both radiation and free convection heat transfer. For the studied situation, they show that, in a pebble bed modular reactor, more than 70% of heat is removed by radiation heat transfer when CO₂ is used as the coolant gas.

Key words: heat transfer, porous medium, pebble bed modular reactor, radiation, free convection, gas, cooling, carbon dioxide

INTRODUCTION

Radiation heat transfer can be found in many industrial applications. In the nuclear field, this mode of heat transfer is encountered in high temperature gas reactors (HTGR). Pebble bed modular reactors (PBMR) fall into the category of HTGR. In normal situations, the removal of heat from a PBMR's core is ensured by a gas flowing through fuel spheres by forced convection, while in accidental situations this is done by free convection. If a tri-atomic gas is used, in some accidental situations, heat can be removed by both radiation and free convection.

The best analytical models for studying heat transfer in PBMR are those developed for the porous media theory. Among the first analytical works done on PBMR by considering a porous medium equation setup is that of Dzung [1] and one of its findings was that the maximum gas temperature is much higher than the mixed mean value. After this, more studies dealing with thermal hydraulics of PBMR were carried out. Stroh *et al.* [2] pre-

sented a mathematical model for the analysis of coupled thermal-hydraulic problems in steady-state pebble bed nuclear reactor cores. None of the usual simplifying assumptions, such as constant properties, constant velocity flow or negligible conduction and/or radiation, were used. Passive heat removal from a PBMR core was studied by Kugeler *et al.* [3] and they deduced that the reduction of the maximum accident temperature below 1600 °C is necessary and can be done in a realistic way by a passive, natural heat transfer mechanism. Most works on heat transfer in PBMR neglect the radiation heat transfer, as is the case with that of Achenbach [4], where the state-of-the-art of heat and flow characteristics of packed beds were presented and new experimental data of heat transfer reported. A number of PBMR thermal-hydraulic results were also given using computer codes or CFD techniques, as in the works done by Dudley [5] and Kim [6]. Because the PBMR core is filled with spheres, it can be studied as a porous medium and all the results obtained can be applicable to a PBMR core. Numerous works dealing with heat transfer exist in the porous media field. Convection heat transfer in vertical cylindrical annuli filled with a porous medium has been studied by

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Havstad and Burns [7]. They proposed an asymptotic solution valid for very tall cylinders and all temperature differences. They also presented some Nusselt results, according to the reactor core aspect ratio and radius ratio. Combined radiation and forced convection heat transfer in porous media was investigated by Talukdar *et al.* [8]. They used a discrete transfer method to solve the radiative part of the energy equation and found that radiation has a significant effect on various parameters studied. Free convection and radiation was studied for a vertical wall with varying temperatures embedded in a porous medium by Badruddin *et al.* [9]. They presented numerical results for the local Nusselt number concerning cases both with and without radiation. In another work, Badruddin *et al.* [10] evaluated radiation and free convection heat transfer through a vertical annulus embedded in a porous medium. They showed the influence of aspect ratio and radius ratio on the Nusselt number, as done by Havstad and Burns [7] for convection heat transfer and commented on the effect of radiation on heat transfer behavior.

Most works do not take into account radiation heat transfer between the cooling gas and the solid surfaces and consider only forced convection, due to the fact that the cooling gas is mono-atomic. In the present work, a tri-atomic gas is used as the coolant for the PBMR core and the study is done in a situation where the CO₂ gas flows solely by its density difference. Under these circumstances, only free convection and radiation heat transfer take place in the PBMR core. In this study, radiation heat transfer occurs between the sphere surfaces and CO₂ in a porous medium in the annular part of the nuclear reactor core. The ultimate aim of our work is to evaluate the rate of radiation heat transfer in comparison to the total heat transfer in a PBMR core.

MATHEMATICAL FORMULATION

This study deals with radiation heat transfer in a porous medium where a tri-atomic gas is involved. As said, it is typically the case of thermal hydraulics in a PBMR where a gas is used to remove the heat produced in the nuclear reactor core. Figure 1(a) shows the gas flow in a PBMR core in a normal situation, while fig. 1(b) depicts an equivalent porous medium in an annular volume. The flow in fig. 1(b) is upward and governed solely by the difference in gas density. This configuration is the case studied and presented in this work. More technological PBMR details are to be found in Singh [11].

The assumptions adopted consider a stationary regime, two dimensions in space and one phase flow. The thermal equilibrium between the fluid and the solid phase is also supposed, performed for a refraction number chosen to be equal to 1, as defined by Bousri *et al.* [12]. The equations governing the flow of an incompressible and inviscid fluid in an annular vertical cylinder filled with a reactive porous medium are

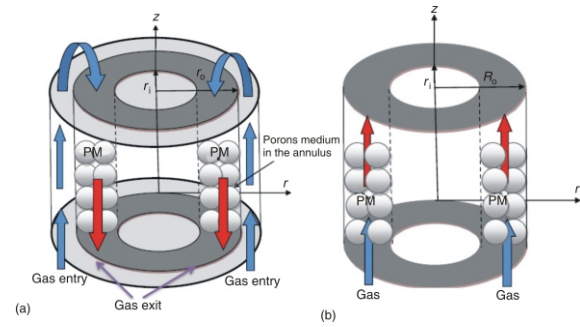


Figure 1. Annular porous medium; (a) porous medium in a PBMR, (b) equivalent porous medium studied

deduced from the principles of the conservation of mass, momentum and energy, as given by Bories *et al.* [13]. The mathematical formulation is established by developing the conservation equations, taking into account the radiation heat flux occurring in annular geometry. This set of equations is written as those defined by Darby [14].

The procedure is a projection of eqs. (1)-(3) in cylindrical coordinates to determine the system of equations governing the heat transfer.

Mass conservation

The equation of mass conservation is written in its general form as follows

$$\frac{\partial \rho}{\partial t} (\rho \bar{U}) = 0 \quad (1)$$

Momentum conservation

The equation of motion, as given by the Darcy model in Kaviany [15], is written as follows

$$\bar{U} = \frac{\kappa}{\mu} (\rho_f g - \rho_m g) \quad (2)$$

Energy conservation: radiation effects considered

In a porous medium, two energy equations are written for both the solid and the fluid phase, as mentioned by Kaviany [15]. At thermal equilibrium, the final energy equation is the sum of these two energy equations. The last term in eq. (3) is added to account for the radiation heat transfer.

$$(\rho_m C_{pm})^* \frac{\partial T}{\partial t} + (\rho_f C_{pf}) \bar{U} T = (\lambda^* T) - \bar{q}_r \quad (3)$$

where ρ_m and ρ_f are the medium and fluid densities, respectively, C_{pm} and C_{pf} – the specific heat at constant pressure of the medium and the fluid. The first term in eq. (3) is given by $(\rho_m C_{pm})^* = (1 - \varepsilon)(\rho_s C_{ps}) + \varepsilon(\rho_f C_{pf})$, where ε is the porosity of the medium, ρ_s and C_{ps} are,

respectively, the solid density and specific heat at a constant pressure of the solid. The subscripts “m”, “s”, and “f” refer, respectively, to the medium, solid phase and the fluid phase. $\vec{U}$ is the velocity vector of the fluid, T – the medium temperature, and λ^* – the effective thermal conductivity of the medium given by $\lambda^* = (1 - \varepsilon)\lambda_s - \varepsilon\lambda_f$, where λ_s and λ_f are, respectively, the thermal conductivity of the solid and the fluid phase. The last term $\vec{q}_r$ represents the gradient vector of the total radiation heat flux density, expressed by

$$\vec{q}_r(s, \Delta) = \int_0^\infty \int_0^{4\pi} L_{\Omega v_c}(s, \Delta) \Delta d\Omega dv_c \quad (4)$$

where $L_{\Omega v_c}(s, \Delta)$ is the spectral directional radiance at the co-ordinate point s in direction Δ and the solid angle $d\Omega$, for frequency v_c .

To find out $L_{\Omega v_c}$ in eq. (4), it is essential to solve the equation defined by Bories *et al.* [13]

$$\frac{1}{\beta_{v_c}} \frac{dL_{\Omega v_c}(s, \Delta)}{ds} = L_{\Omega v_c}(s, \Delta) (1 - \omega_{v_c}) L_{v_c}^0 [T(s)] - \frac{\omega_{v_c}}{4\pi} \int_0^{4\pi} p_{v_c}(\Delta, \Delta') L_{\Omega v_c}(s, \Delta') d\Delta' \quad (5)$$

where $\beta_{v_c} = \chi_{v_c} / \sigma_{v_c}$ is the extinction coefficient, χ_{v_c} is the absorption coefficient, σ_{v_c} – the diffusion coefficient as $1 / \omega_{v_c} = \chi_{v_c} / \beta_{v_c}$, $\omega_{v_c} = \sigma_{v_c} / \beta_{v_c}$ – the albedo $L_{v_c}^0 [T(s)]$ – the monochromatic radiance of the blackbody and is calculated by

$$L_{v_c}^0 [T(s)] = \frac{2h\nu_c^3}{C_{v_c}^2 \exp \frac{h\nu_c}{k_B T} - 1}$$

The radiation in the environment is $c_{v_c} = C_0 / n_{v_c}$, where the wave velocity in the vacuum is given by: $C_0 = 299\,792\,458$ m/s, n_{v_c} – the refractive index and p_{v_c} – the phase function, while $h = 6.626 \cdot 10^{-34}$ J/s – the Planck constant and $k_B = 1.381 \cdot 10^{-23}$ J/K – the Boltzmann constant. $p_{v_c}(\Delta, \Delta')$ is the phase function or diffusion indices in the medium. It is interpreted physically as the ratio of the intensity of scattered radiation in a direction by the intensity of the radiation scattered if the diffusion is isotropic in the same direction as defined by Kamdem [16] then

$$\frac{1}{4\pi} \int_0^{4\pi} p_{v_c}(\Delta, \Delta') d\Delta' = 1 \quad (6)$$

When integrating eq. (5) over the entire solid angle Ω , from 0 to 4π , taking into account eq. (6), according to Kamdem [16], we get

$$\frac{4\pi dL_{v_c}(s, \Delta)}{ds} d\Omega = \beta_{v_c} \int_0^{4\pi} L_{v_c}(s, \Delta) d\Omega + 4\pi \chi_{v_c} L_{v_c}^0 [T(s)] - \sigma_{v_c} \quad (7a)$$

$$\text{We have } \int_0^{4\pi} L_{v_c}(s, \Delta) d\Delta' = \int_0^{4\pi} L_{v_c}(s, \Delta) d\Omega$$

Equation (7a) is transformed as

$$\frac{4\pi dL_{v_c}(s, \Delta)}{ds} d\Omega = 4\pi \chi_{v_c} L_{v_c}^0 [T(s)] - \sigma_{v_c} \int_0^{4\pi} L_{v_c}(s, \Delta) d\Delta' \quad (7b)$$

and, taking into account eq. (4), $\vec{q}_r$ is written as follows

$$\vec{q}_r = \int_0^\infty \int_0^{4\pi} L_{v_c}(s, \Delta) \Delta d\Omega dv_c = \int_0^\infty 4\pi \chi_{v_c} L_{v_c}^0 [T(s)] dv_c - \int_0^\infty \int_0^{4\pi} \chi_{v_c} L_{v_c}(s, \Delta) \Delta d\Omega dv_c \quad (8)$$

Figure 2 represents the projection of the radiance describing the radiation of the solid particle S in a direction through a solid angle Ω in the radial direction r , axial direction z and angular direction θ .

Figure 3 represents the projection of the solid angle Ω according to space co-ordinates r , z , and θ and the definition of the curvilinear derivative in space direction z . Direction Δ is defined according to space co-ordinates (r, z, θ) with, respectively: $\sin \theta \cos \phi$ along r , $\sin \theta \sin \phi$ along z , and $\cos \theta$ following θ , as expressed by Kamdem [16].

The variation of the solid angle is given by

$$d\Omega = \sin \theta d\theta d\phi \quad (9)$$

where, θ and ϕ are, respectively, the polar and the azimuthal solid angle Ω .

According to Kamdem [16]: $m = \cos \theta$ and its derivative is $dm = -\sin \theta$ and when replaced in eq. (9), one obtains

$$d\Omega = -dm d\phi \quad (10)$$

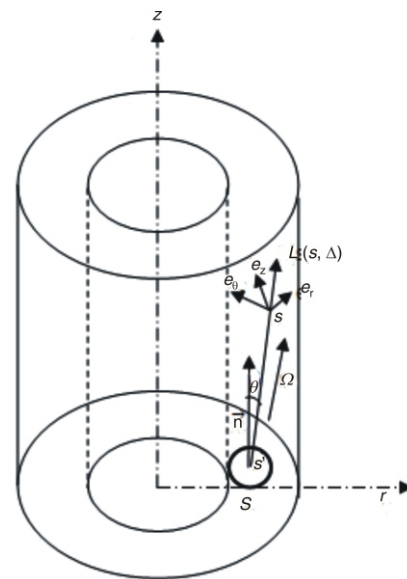


Figure 2. Projections of the radiance according to the solid angle

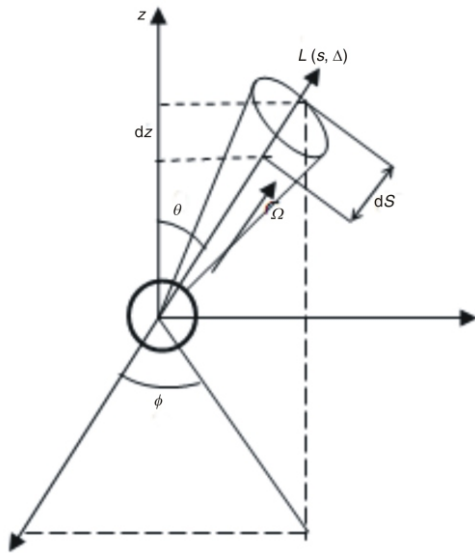


Figure 3. Projection of the solid angle

Now, considering a surface in plane (r, θ) and the radiation heat transfer taking place only in the z direction, the curvilinear derivatives are given by: $d/ds = \partial/\partial z \, dz/ds$

With $dz/ds = m$, one can write

$$\frac{d}{ds} = m \frac{\partial}{\partial z} \quad (11)$$

Taking into account eq. (11), the radiation balance eq. (5) leads to

$$\begin{aligned} 4\pi \frac{\partial L_{v_c}(s, \Delta)}{\partial z} d\Omega - \beta_{v_c} \int_0^{4\pi} L_{v_c}(m, \Delta) d\Omega = \\ 4\pi \chi_{v_c} L_{v_c}^0 [T(s)] \alpha_{v_c} \int_0^{4\pi} L_{v_c}(m, \Delta) d\Omega \quad (12) \end{aligned}$$

It is considered herein that the spheres are randomly dispersed in the medium and that the radiation properties are independent of the azimuth so that, consequently, one can write

$$\begin{aligned} L_{v_c}(s, \Delta) &= L_{v_c}(z, m) \\ \int_0^{4\pi} d\Omega &= 2\pi \int_0^1 dm \\ p(\Delta, \Delta) &= p(m, m) \end{aligned}$$

Let us consider a sphere surface element, as presented in fig. (3), defined by r , θ , and z . Radiation heat transfer occurs only in the direction of the vector normal to this surface and thus the vector gradient of the radiation heat flux density is written as: $\vec{q}_r = \partial \vec{q}_r / \partial z$ and the eq. (8) becomes

$$\begin{aligned} \vec{q}_r = \int_0^\infty 4\pi \chi_{v_c} L_{v_c}^0 [T(s)] dv_c \\ \int_0^\infty \int_0^1 L_{v_c}(z, m) dm dv_c \quad (13) \end{aligned}$$

while energy conservation eq. (3), when taking into account radiation heat transfer, becomes

$$\begin{aligned} (\rho C_p)_f u \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} - \frac{\lambda^*}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) - \frac{\partial}{\partial z} \left(\lambda^* \frac{\partial T}{\partial z} \right) \\ = \int_0^\infty 4\pi \chi_{v_c} L_{v_c}^0 [T(s)] dv_c - \int_0^\infty \int_0^1 \chi_{v_c} \frac{2\pi}{L_{v_c}(z, m)} dm dv_c \quad (14) \end{aligned}$$

Conservation equations projected in (r, z) plane are finally written as

$$\frac{1}{r} \frac{\partial(u, r)}{\partial r} - \frac{\partial w}{\partial z} = 0 \quad (15)$$

$$u = \frac{K}{\mu} \frac{\partial p}{\partial r}, \quad w = \frac{K}{\mu} \frac{\partial p}{\partial z} - pg \quad (16)$$

$$\begin{aligned} (\rho C_p)_f u \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} - \frac{\lambda^*}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) - \frac{\partial}{\partial z} \left(\lambda^* \frac{\partial T}{\partial z} \right) \\ = \int_0^\infty 4\pi \chi_{v_c} L_{v_c}^0 [T(s)] dv_c - \int_0^\infty \int_0^1 \chi_{v_c} \frac{2\pi}{L_{v_c}(z, m)} dm dv_c \quad (17) \end{aligned}$$

The integral form of the right term in eq. (17) represents the expression of the radiation heat flux.

According to Boulet [17], integrals in eq. (17) can be replaced by sums as follows

$$\begin{aligned} \int_0^\infty 4\pi \chi_{v_c} L_{v_c}^0 [T(s)] dv_c = \sum_{v_c} \int_0^\infty \frac{2\pi}{L_{v_c}(z, m)} dm dv_c \\ = \sum_{v_c} 4\pi \chi_{v_c} L_{v_c}^0(T) \Delta v_c \sum_m \frac{2\pi}{L_{v_c}(z, m)} \Delta m \Delta v_c \quad (18) \end{aligned}$$

So, eq. (17) can be written as

$$\begin{aligned} (\rho C_p)_f u \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} - \frac{\lambda^*}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) - \frac{\partial}{\partial z} \left(\lambda^* \frac{\partial T}{\partial z} \right) \\ = \sum_i^n 4\pi \chi_i L_i^0(T) \frac{1}{2} \sum_j^M L(z, m) \Delta m_j \Delta v_{c_i} \quad (19) \end{aligned}$$

The flow model adopted for the porous medium is the Darcy one. Conservation equations are given in dimensionless forms, reporting the variables with reference to variables such as outer radius r_e , velocity U_e and temperature T_e of the fluid. According to Bories *et al.* [13], if the following dimensionless quantities are considered

$$\begin{aligned} \dot{u} = \frac{u}{\alpha / r_e}, \quad \dot{w} = \frac{w}{\alpha / r_e}, \quad P = \frac{P}{\alpha \mu / K}, \quad \frac{\rho_e g z}{\alpha \mu / K}, \\ \tau = \frac{T - T_e}{T_i - T_e}, \quad \dot{z} = \frac{z}{r_e}, \quad \dot{r} = \frac{r}{r_e}, \quad L = \frac{L}{\sigma T_e^4} \end{aligned}$$

then the mass, momentum, and energy conservation equations are written, respectively, as

$$\frac{\partial(\dot{u} \dot{r})}{\partial \dot{r}} - \dot{r} \dot{r} \frac{\partial \dot{w}}{\partial \dot{z}} = 0 \quad (20)$$

$$\frac{\partial \dot{u}}{\partial \dot{z}} - \frac{\partial \dot{w}}{\partial \dot{r}} = R_a^* \frac{\partial \tau}{\partial \dot{r}} \quad (21)$$

$$\begin{aligned} \dot{u} \frac{\partial \tau}{\partial \dot{r}} - \dot{w} \frac{\partial \tau}{\partial \dot{z}} - \frac{1}{\dot{r}} \frac{\partial \tau}{\partial \dot{r}} - \frac{\partial^2 \tau}{\partial \dot{r}^2} - \frac{\partial^2 \tau}{\partial \dot{z}^2} \\ - \frac{4\sigma T_e^4 r_e^2 (1 - \omega) \Delta T}{\lambda^* \beta} - \pi \dot{L}^0(\tau) - \frac{\pi}{2} \frac{L(z, m) m}{v_c} \end{aligned} \quad (22)$$

where, $\dot{u}$ and $\dot{w}$ are the dimensionless forms of velocity components, $\dot{r}$ and $\dot{z}$ – the dimensionless forms of spatial co-ordinates, τ is the dimensionless form of the medium temperature, $\dot{L}$ – the dimensionless form of radiance, K – the permeability, α – the thermal diffusivity, μ – the dynamic viscosity, β – the expansion coefficient, ω – the albedo, g – the gravity acceleration, ρ – density, T – the temperature difference, r_e – the outer radius of the cylinder $m = \cos \phi$, and $Ra_m = Kr_e g \rho_c \beta / \alpha \mu$ is the modified Rayleigh number, as given by Incropera [18], defined as being the product of the Rayleigh number $Ra = g \beta \Delta T r_e^3 / \nu \alpha$ and the Darcy number $Da = K / r_e^2$. The expansion coefficient β is calculated, as proposed by Serth [19], assuming $T_e = (800 + 273) \text{ K}$, by $\beta = 1/T_e = 0.0009319 \text{ K}^{-1}$, while the pebble bed permeability is evaluated by $K = d^2 (\varepsilon^{5.5}/5.6)$.

We note the apparition of the coupled parameter as the one calculated for conduction-radiation by Kamdem [16]. This coupling parameter is the inverse ratio of the quantities given in the term sum in eq. (22), noted as $N = \lambda^* \beta / 4\sigma T_e^4$. In summary, one can write the following simple equation

$$Q_v = \frac{4\omega r_e^2}{N} Q_r \quad (23)$$

where, Q_v and Q_r are, respectively, the convective and radiative heat transfer.

Boundary conditions

The inner wall of the bed region is adiabatic (eq. 24), while the outer one exchanges heat with the surroundings (eq. 25). The Biot number defines the conduction thermal resistance on the convective one. In this study one focused on Biot numbers greater than unity, such as the values taken to be 5, 10, 30, 50, and 70, and presented in tab. (6) because, the conduction is slow in the sphere body, then at its surface, the temperature gradients in the sphere body are not negligible and the thermal resistance is localized at the sphere side. These Biot values are much closer to the real theoretical behavior of a PBMR. The case where Biot number = 0.5 is taken to establish the effect of thermal

resistance, when it is localized in the fluid, on the total heat transfer.

The radius ratio is defined as $\eta = r_i/r_e$, the height ratio by $A = H/r_e$ and, therefore, $\dot{r}$ varies from η to 1 and $\dot{z}$ varies from 0 to A .

– At the inner wall of the annulus

$$\dot{r} = \frac{r_i}{r_e} \quad \tau = 1 \quad (24)$$

– At the outer wall of the annulus

$$\dot{r} = 1 \quad \frac{\partial \tau}{\partial \dot{r}} = Bi \tau \quad (25)$$

– At the inlet of the annulus

$$\dot{z} = A \quad \frac{\partial \tau}{\partial \dot{z}} = 0 \quad (26)$$

– At the outlet of the annulus

$$\dot{z} = 0 \quad \frac{\partial \tau}{\partial \dot{z}} = 0 \quad (27)$$

A stream function is introduced to solve the system of eqs. (20)-(22), giving the convective heat transfer Q_v . In order to calculate the radial velocity component, one has to consider the temperature expression appearing in the heat transfer equation. This method is well explained by Havstad and Burns [7]. After calculations, a dimensionless form of the convective part is found

$$\begin{aligned} Q_v = \frac{2\gamma Ra Bi \pi}{(1 - Bi \ln \eta)} \\ \frac{\eta^2 \ln \eta - \eta^2 / 2 - 1/2}{2\eta^2 - 1} (1 - Bi \eta^2 - Bi \eta^3 \ln \eta - Bi \eta^3) \\ - Bi \frac{1}{2} - Bi \eta^3 - Bi \eta^3 \ln \eta - \frac{\eta^3 \ln \eta}{2} - \frac{\eta^3}{2} \end{aligned} \quad (28)$$

where $\gamma = 0.57$ is a proportionality constant given by Havstad and Burns [7].

The dimensionless total radiation heat flux is written according to Kamdem [16] as

$$Q_r = 2\pi \int_0^1 \int_0^1 \dot{L}(z, m) dm dv_c \quad (29)$$

Taking into account thermal equilibrium, radiation heat transfer and constant isotropic radiation, one can establish a simple formula between radiance and the emitted radiation $L = M/\pi$ known as Lambert's law, with M given by $M = e_s \sigma (T_s^4 - T_i^4)$. So, this allows replacing the integral in eq. (29) by a sum. The radiation heat flux becomes

$$Q_r = 2 \sum_{v, m} e_s \sigma (T_s^4 - T_i^4) \Delta m \Delta v_c \quad (30)$$

where e_s is the carbon emissivity, σ – the Stephan-Boltzmann constant, T_s – the sphere surface temperature, T_i the CO_2 inlet temperature, m – the value given by $\cos \theta$, and v_c – the frequency.

The total heat flux can now be evaluated thanks to eqs. (23), (28), and (30). It is expressed by the sum of the convective part and the radiative part

$$Q_r = Q_v = Q_r = N_v q_v = Q_r = \frac{r_e^2}{\alpha \sigma T_e^4} q_v = Q_r \quad (31)$$

where $N_v = r_e^2 / \alpha \sigma T_e^4$ is the number that relates the radiation to the convection in the global thermal eq. (28).

The PBMR geometrical characteristics are taken from Venter *et al.* [20]. The generated power, the pressure of the system, the inlet and outlet gas temperature difference and the CO₂ mass flow rate are maintained the same as when using helium gas. These quantities are summarized in tab. 1

Table 1. PBMR characteristics

Parameters	Identity
Generated power	400 MW
CO ₂ inlet temperature	400 °C
CO ₂ outlet temperature	800 °C
Pressure of the system	9 MPa
CO ₂ mass flow	192 kg/s
PBMR core outside diameter	3.7 m
PBMR core inside diameter	2.0 m
Fuel spheres number	450000
Fuel sphere diameter	0.06 m
Enrobing fuel material	Carbon

The considered study data taken from Incropera *et al.* [18] are presented in tab. (2). The physical properties are calculated for an arithmetic averaged temperature between the entry and the exit of the gas.

Venter *et al.* [20] used a Computer Aided Design (CAD) to create the complex geometry of the reactor core in 3-D and to extract accurate geometric information such as volumes. In our work, this is not the case; the annular volume is developed in a Cartesian system to render a cubic volume. The volume is calculated, for an inline disposition of spheres, by multiplying the height by width by length: $50d \times 82d \times 110d$, respectively, which gives exactly $451000 d^3 \text{ m}^3$. Where d is the sphere diameter. This volume has been rounded to $450000 d^3 \text{ m}^3$. We consider the disposition of the

Table 2. Considered study data from Incropera *et al.* [18]

Quantity	Value
Temperature at the surface of the sphere [°C]	1000
Carbone emissivity	0.98
CO ₂ inlet temperature [°C]	400
CO ₂ cinematic viscosity [m^2s^{-1}]	$8.424 \cdot 10^{-5}$
CO ₂ diffusion coefficient [m^2s^{-1}]	$1.153 \cdot 10^{-4}$
Gravity [ms^{-2}]	9.81
CO ₂ outlet temperature [°C]	800
Stephan-Boltzmann constant [$\text{Wm}^{-2}\text{k}^{-4}$]	$5.67 \cdot 10^{-8}$

Table 3. Application results

Quantity	Equation	Value
Total volume [m^3]	$V_t = 450000 d^3$	97.2
Volume of the solid phase [m^3]	$V_s = (4/3) (d/2)^3 \cdot 450000$	50.8938
Volume of the fluid phase [m^3]	$V_f = V_t - V_s$	46.3062
Porosity	$\varepsilon_f = V_f / V_t$	0.4764
Permeability [m^2]	$K = \frac{\varepsilon_f^{5.5}}{5.6} d^2$	$1.08883 \cdot 10^{-5}$

spheres in the annulus of the PBMR core as an inline disposition.

Some porous medium specific quantities are calculated and presented in tab. 3. It must be stressed that all results produced are directly dependent on the calculated porosity.

In tab. 4, in order to calculate the radius ratio $\eta = r_i / r_e$ and evaluate the modified Rayleigh number, the values for the inner and outer radius of the PBMR core are given.

Calculations have to be done considering a constant core volume and a constant number of spheres. Any changes in the core height will automatically induce a change in the core radius and *vice versa*.

In tab. 5, the values for core reactor height H are given. The values of the outer radius r_e (tab. 4) are used to calculate ratio $A = H / r_e$. This part of the work restricts parametric studies in order to always keep the core volume and the number of spheres constant.

The values of the Biot number used in the calculations are reported in tab. 6.

The values of the coupling parameter N_v , presented in tab. 7, are calculated by varying the outer radius of the nuclear reactor core from 3.7 m to 8.7 m.

Table 4. Rayleigh number calculation for the given radii ratio

η	r_e [m]	r_i [m]	Ra_m
0.54	3.7	2.0	15153.0312
0.63	4.7	3.0	19248.4451
0.7	5.7	4.0	23343.8589
0.74	6.7	5.0	27439.2727
0.77	7.7	6.0	31534.6866
0.8	8.7	7.0	35630.1004

Table 5. Form factor and height values

A	0.5	1	2	5	10	20
H	1.85	4.7	11.4	33.5	77	174

Table 6. Biot number values

Bi	0.5	5	10	30	50	70
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Table 7. Coupling parameter values

r_e [m]	3.7	4.7	5.7	6.7	7.7	8.7
N_v	1.5797639	2.5490858	3.7491987	5.18010248	6.84179719	8.73428283

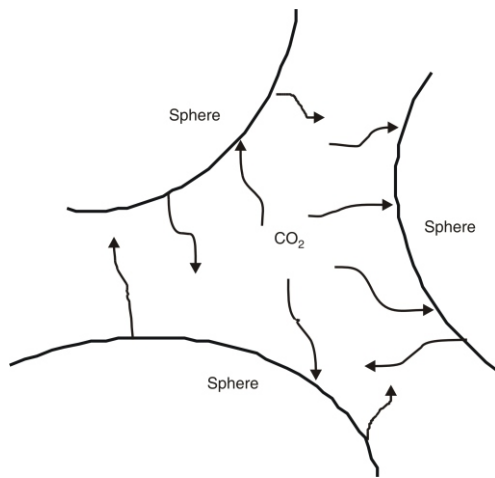
Table 8. Values of frequency, polar angle and m

F	6	8	10	12	14	16
θ°	0	15	30	45	60	85
$m = \cos \theta$	1	0.96596017	0.86615809	0.70738827	0.50045969	0.08790494

The frequency F , the polar angle θ , and the $\cos(\theta)$ values used to evaluate the dimensionless total radiation heat flux in eq. (30) are presented in tab. 8.

Radiation heat transfer calculations

The radiation heat transfer in a PBMR occurs between a gray surface represented by the carbon that covers the spherical fuel pellets and the gas, in this case CO_2 , used as coolant, as shown in fig. 4. A model describing this heat transfer was proposed by Jannot [21]. To describe the radiation heat transfer occurring in a PBMR core, the following governing equations are established and the calculating flowchart is presented in fig. 5. In fig. 4, only the mechanism of radiation heat transfer between the spheres and the gas is shown. The solid and gas heat conduction are taken into account in eq. (3).

**Figure 4. Illustration of radiation heat transfer between CO_2 and the solid surfaces**

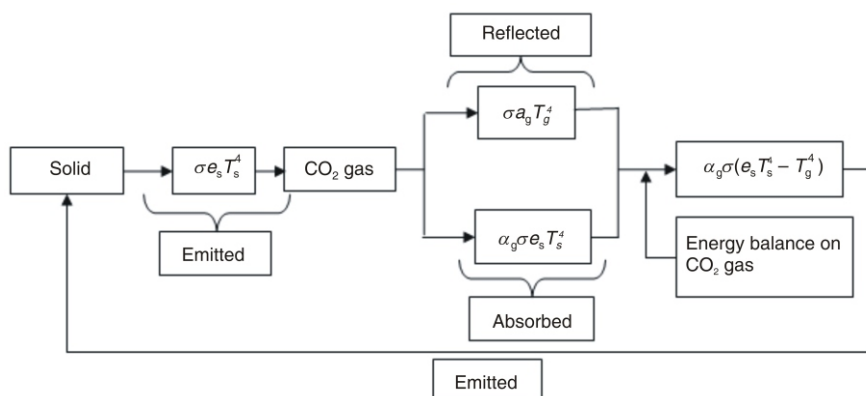
Carbon emits to the gas a radiation density equal to $\sigma e_s T_s^4$, where e_s is the carbon emission factor. The gas reflects an amount of the radiation heat flux equivalent to $\sigma e_g T_g^4$ and absorbs a quantity of $\alpha_s \sigma e_g T_g^4$, where α_s is the absorption factor. After balancing, the net flux density obtained is

$$Q_{\text{net}} = \sigma a_s T_s^4 - \alpha_s \sigma e_g T_g^4 = a_s \sigma (e_s T_s^4 - T_g^4) \quad (32)$$

RESULTS AND DISCUSSION

Variations of the total heat flux recovered by the CO_2 gas are presented. Firstly, calculations are done regardless of the radiation heat transfer. Secondly, calculations are done taking into account the radiation heat transfer. A comparison between the two situations is then made. The presented results concern the variations of the total heat flux as a function of the radius ratio varying from 0.54 to 8.0, for the given different Biot number values between 0.5 and 70, and for different Rayleigh number values.

Presented in fig. 6 the dimensionless total heat flux variations, without considering the radiation heat transfer, as a function of the radius ratio varying between 0.54 and 0.8 and the different values of the Biot number between 0.5 and 70 and for a fixed Rayleigh number equal to 15153. One can note that, for the small values of the radius ratio ranging from 0.54 to 0.63, the heat flux takes values between 0 and 200000. For a Biot number equal to 5 and 10, the heat flux varies between 100,000 and 500,000; but, for the large values of the Biot between 30 and 70, the heat transfer increases with the radius ratio and becomes more important, ranging from 150,000 to 1,000,000. It can also be noted that for a Biot number equal to 0.5, the heat flux is almost zero. One observes the same variations of the heat flux as a function of the radius ratio for very large Biot number values.

**Figure 5. Radiation heat transfer flowchart**

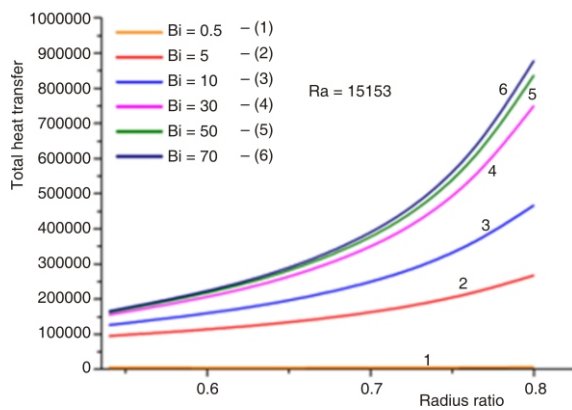


Figure 6. Dimensionless variations of the total heat flux regardless of radiation heat transfer as a function of the radius ratio for different values of the Biot number and a constant Rayleigh number equal to 15153

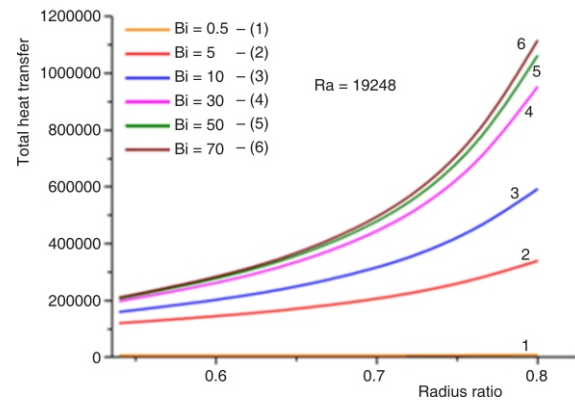


Figure 8. Dimensionless variations of the total heat transfer as a function of the radius ratio for different values of the Biot number and for a Rayleigh number equal to 19248, regardless of the radiation heat transfer

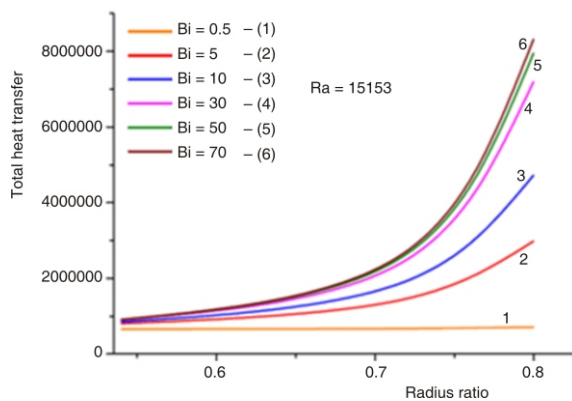


Figure 7. Dimensionless heat flux variations depending on the total radius ratio for different values of the Biot number and a number of the Rayleigh constant equal to 15153, taking into account radiation heat transfer

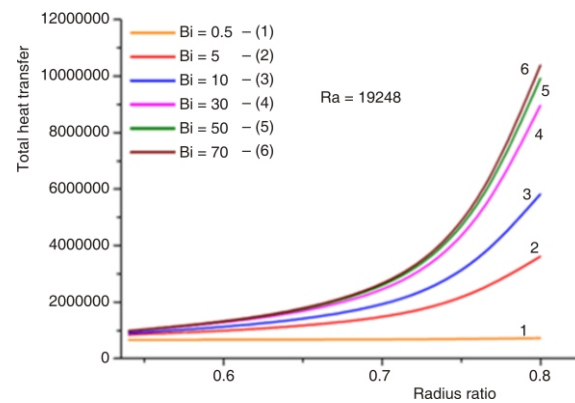


Figure 9. Dimensionless heat transfer variations depending on the radius ration for different values of the Biot number of a Rayleigh number equal to 19248, taking into account radiation effects

Presented in fig. 7 the dimensionless variations of the total heat flux according to the radius ratio between 0.54 and 0.8, for different values of the Biot number and for a constant value of the Rayleigh number equal to 15153, taking into account the radiation heat transfer. One can note that for each value of the Biot number, for small radius ratios less than 0.55, the heat flux takes almost identical values and the flux increases according to the radius ratio, except for a Biot number equal to 0.5 where it remains constant. It can also be noted that the heat flux has identical variations as a function of the radius ratio for large values of the Biot number.

Figure 8 shows the dimensionless heat transfer variations for different values of the radius ratio between 0.54 and 0.8, and for different values of the Biot number and constant value of the Rayleigh number equal to 19248, regardless of the radiation heat flux. For all values of the Biot number and for small radius ratios less than 0.55, the heat flux varies between 0 and 200000. Upon this, the heat flux increases with the increase in the radius ratio for all Biot numbers, except for those equal to 0.5, where it remains constant.

Heat flux variations are plotted in fig. 9, according to the radius ratio for the values of the Biot number between 0.5 and 70 and a constant Rayleigh number equal to 19248, taking into account radiation heat transfer. One notices that for a Biot number equal to 0.5, the heat flux is low and almost constant. It can also be noted that for Biot numbers between 5 and 70, for small values of the radius ratio less than 0.55, the heat flux varies between 0 and $1.5 \cdot 10^7$, after which the heat flux increases significantly, along with the increase in the radius ratio.

When comparing figs. 6 and 7 and figs. 8 and 9, one can see that when taking into account the radiation heat transfer, the dimensionless total heat flux evacuated from the PBMR core increases and becomes more important.

CONCLUSIONS

This work shows that the radiation heat transfer plays a major role in the extraction of heat from a

PBMR core, even more so if forced convection is avoided. Applied to a PMBR, the heat flux through an annular geometry filled with a porous medium depends on several parameters, such as the ratio of the inner and outer radius of the annulus core (as discussed by Havstad and Burns [7], as well as by Badruddin *et al.* [10]), and the modified Rayleigh and Biot numbers. Obtained results show that the total heat flux removed from a PBMR core, with or without the consideration of radiation heat transfer, increases when the Biot number increases, as well as upon an increase in the modified Rayleigh number. On the other hand, if taking into account the radiation heat transfer, this significantly increases the total heat flux recovered by the coolant. This study shows that, for a porosity of 0.47, when using CO₂ as a coolant in a PBMR, the heat removed by the radiation heat transfer amounts to about 70% to 80 % of the total removed heat. This radiation heat transfer represents more than eight times the heat removed by free convection, making CO₂ a good gas coolant in post shutdown or emergency situations. This technique of cooling the PBMR core may be refined with time, after the porous media configuration undergoes an upgrade.

AUHTOR CONTRIBUTIONS

Theoretical analysis was carried out by all authors. All authors analysed and discussed the results. The manuscript was written by T. Hassani, K. Sidi-Ali, and K. Oukil, and the figures were prepared by Y. Amri and T. Hassani.

REFERENCES

- [1] Dzung, L. S., A Cooling Problem of Pebble Bed Nuclear Reactors, *Int. J. Heat Mass Transfer*, 1 (1960), 2-3, pp. 236-241
- [2] Stroh, K. R., Jiacoletti, R. J., Olson, H. J., Thermal-Hydraulic Analysis Techniques for Axisymmetric Pebble Bed Reactor Cores, *Nuclear Engineering and Design*, 52 (1979), 3, pp. 343-347
- [3] Kugeler, K., Philippen, P. W., Schmidtlein, P., Swatoh, R., Passive Heat Removal from the Core of Small and Medium Sized Pebble Bed Reactors, *Nuclear Engineering and Design*, 136 (1992), 1-2, pp. 143-148
- [4] Achenbach, E., Heat and Flow Characteristics of Packed Beds, *Experimental Thermal and Fluid Science*, 10 (1995), 1, pp. 17-27
- [5] Dudley, T., *et al.*, The Thermal-Hydraulic Model for the Pebble Bed Modular Reactor (PBMR) Plant Operator Training Simulator System, *Nuclear Engineering and Design*, 238 (2008), 11, pp. 3102-3113
- [6] Kim, H., Lee, J. I., No, H. C., Thermal Hydraulic Behavior in the Deteriorated Turbulent Heat Transfer Regime for a Gas-Cooled Reactor, *Nuclear Engineering and Design*, 240 (2010), 4, pp. 783-795
- [7] Havstad, M. A., Burns, P. J., Convective Heat Transfer in Vertical Cylindrical Annuli Filled with a Porous Medium, *Int. J. Heat Mass Transfer*, 25 (1982), 11, pp. 1755-1766
- [8] Talukdar, P., *et al.*, Combined Radiation and Convection Heat Transfer in a Porous Channel Bounded by Isothermal Parallel Plates, *International Journal of Heat and Mass Transfer*, 47 (2004), 5, pp. 1001-1013
- [9] Badruddin, I. A., *et al.*, Free Convection and Radiation for a Vertical Wall with Varying Temperature Embedded in a Porous Medium, *Int. J. of Thermal Sciences*, 45 (2006), 5, pp. 487-493
- [10] Badruddin, I. A., *et al.*, Heat Transfer by Radiation and Natural Convection through a Vertical Annulus Embedded in Porous Medium., *Int. Communications in Heat and Mass Transfer*, 33 (2006), 4, pp. 500-507
- [11] Singh, J., Hohn, H., Barnert, H., Advanced Pebble Bed High Temperature Reactor with Central Graphite Column for Future Applications, *Nuclear Engineering and Design*, 128 (1991), 3, pp. 383-390
- [12] Bousri, A., Bouhadeh, Kh., Numerical Study of Heat and Mass Transfer in a Cylinder Filled with a Porous Medium, Original title in French: Etude Numérique du Transfert de Chaleur et de Masse Dans un Cylindre Rempli d'un Milieu Poreux, Euro therm Seminar N°81 Reactive Heat Transfer in Porous Media, Albi, France, June 4-6, 2007
- [13] Bories, S., Prat, M., Heat Transfer in Porous Media; Original title in French: Transfert de Chaleur Dans les Milieux Poreux, Techniques de l'Ingénieur, Traité Génie Energétique, B8250, pp. 1-33, 1994
- [14] Darby, R., Chemical Engineering Fluid Mechanics, 2nd edition, Dekker, New York, 2001
- [15] Kaviany, M., Principles of Heat Transfer in Porous Media, 2nd ed, Springer-Verlag, New York, 1995
- [16] KamdemTagne, H. T., Study of Heat Transfer in an Anisotropic Porous Medium, Application to Thermal Insulation in Silica Fiber, Original title in French: Etude du Transfert Thermique Dans un Milieu Poreux Anisotrope, Application Aux Isolants Thermiques en Fibre de Silice, Ph.D. thesis, INSA Lyon, France, 2008
- [17] Boulet, P., Jeandel, G., Morlot, G., Theoretical Study of non Fibred Material Influence on the Radiation Behavior of Fibred Insulators, Original title in French: Etude Theorique de l'Influence des Infibres sur le Comportement Radiatif des Isolants Fibreux, *Int. J. Heat Mass Transfer*, 39 (1996), 15, pp. 3221-3231
- [18] Incropera, F. P., Fundamentals of Heat and Mass Transfer, 6th ed., John Wiley & Sons, New York, USA, 2006
- [19] Serth, R. W., Process Heat Transfer, Principles and Applications, Elsevier Science and Technology Books, 2007
- [20] Venter, P. J., Mitchell, M. N., Integrated Design Approach of the Pebble Bed Modular Reactor Using Models, *Nuclear Engineering and Design*, 237 (2007), pp. 1341-1353
- [21] Jannot, Y., Heat Transfer, Original title in French: Transferts Thermiques, Nancy School of Mines, France, 2008

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**ПРОЦЕНА ПРЕНОСА ТОПЛОТЕ ЗРАЧЕЊЕМ У ПОРОЗНИМ СРЕДИНАМА
Примена на модуларни реактор са сврним горивним елементима хлађен угљендиоксидом**

У раду се анализира допринос преноса топлоте зрачењем, хлађењу модуларног реактора са сврним горивним елементима. Математички модел развијен за порозну средину заснива се на скупу једначина применљивом за кружну геометрију. У претходним важнијим радовима о овом предмету, разматран је модел принудне конвекције, често без урачунавања преноса топлоте зрачењем. Овде се проучава слободна конвекција и пренос топлоте зрачењем, који се јављају током уклањања заостале топлоте после престанка рада или током ванредног догађаја. У намери да се изведу кључне једначине преноса топлоте зрачењем, разматрано је стационарно стање изотропне емисионе средине угљендиоксида. Добијени систем једначина записан је у бездимензијалном облику и потом решен. Ради процене учинка преноса топлоте зрачењем на укупну уклоњену топлоту, систем једначина решен је аналитичким поступком. Резултати омогућују квантификовање преноса топлоте, како зрачењем тако и слободном конвекцијом. У проучаваном примеру модуларног реактора са сврним горивним елементима, показано је да се више од 70% топлоте уклања преносом топлоте зрачењем уколико је угљендиоксид коришћен као хладилац.

Кључне речи: пренос топлоте, порозна средина, модуларни реактор са сврним горивним елементима, зрачење, слободна конвекција, гас, хлађење, угљендиоксид
