APPLICATION OF AN ADDITIVE-TYPE MIXED PROBABILITY DISTRIBUTION TO THE ANALYSIS OF RADIATION FROM A MIXTURE OF RADIOACTIVE SOURCES

by

Diana Ć. DOLIĆANIN-DJEKIĆ [1]***, Dženis F. PUĆIĆ*** [2]***, and Koviljka Dj. STANKOVIĆ*** [3*]

[1]Faculty of Technical Sciences, University of Kosovska Mitrovica, Serbia
[2]State University of Novi Pazar, Novi Pazar, Serbia
[3]Faculty of Electrical Engineering, University of Belgrade, Belgrade, Serbia

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This paper investigates the possibility of distinguishing between the effects of radiation coming from two or more radioactive isotopes, by using methods of statistical mathematics. The procedure uses a mixed distribution of an additive type. Mathematical treatment is demonstrated herein on an example of analysis of composite radiation from two radioactive sources.

Key words: mixture of radioactive isotopes, additive-type mixed probability distribution, parameters for individual distribution universes

INTRODUCTION

Decay time of an individual nucleus fluctuates irregularly from one nucleus to another, so that, at the present level of development of science, it is not possible to predict whether a specific nucleus will decay tomorrow, next week, or in a million years. An individual nucleus decay is considered completely random, without anything to determine the time when it happens, there being no law that would relate it to anything [1, 2]. For this reason, when a mixture of radioactive isotopes is considered, experimentally obtained distribution functions for the random number of decays (*i. e.* the number of pulses in a detector) represent a combination of two or more theoretical distribution functions. Combinations of distributions result in mixed distributions. These can be of additive or multiplicative type, with the letter ones generally pertaining to probability enlargement laws [3-5]. Before one can reach the stage of statistical formulation of a random (stochastic) process, it needs to be "phenomenologically known". One therefore has to start with experiments whose results (number of pulses in a detector) vary within certain random limits. The causes of variations of experimental results can be inherent to the process of decay of radioactive isotopes, arising from its boundary conditions, or may reside in random measuring errors. This last category should be minimized and be determined as accurately as possible, since it can not be covered by the algorithm proposed in this paper [6, 7].

The aim of this paper is to develop an algorithm which enables radiation from a mixture of radioisotopes to be analyzed by a statistical method.

THE EXPERIMENT

Empirical distributions of the random variable designated as "the number of pulses in the detector" for a mixture of sources were obtained by means of a GM counter and two radioactive isotopes, Cs-137 and Am-241. The sources were specifically positioned relative to the GM counter's tube, so that the Am source would provide a pulse count higher by approximately one third than that from the Cs source. The GM counter provided with anti-coincidence protection was placed within a shielded booth (shielding it from electromagnetic radiation) with a suppression level of more than 100 dB. Measurements were performed at five minute intervals, first with the Cs source only, then with the Am source, and finally with both sources. Geometrical parameters of the system, including the position of the sources and the detector, remained unchanged during measurements. Results were corrected for background radiation, which was also measured, although it proved to be negligible. The combined measurement uncertainty of the whole experimental procedure was below 3% [8, 9].

RESULTS AND DISCUSSION

Number of pulses in the detector is a stochastic quantity which follows the normal distribution

* Corresponding author; e-mail: kstankovic@etf.rs

[10, 11]. Figure 1 shows the experimentally obtained distribution functions for the Cs source, F(Cs), for the Am source, F(Am), and for the superposition of Cs and Am sources, F(Cs + Am). Results indicate that the last of these three distributions represents a mixed distribution of the additive type.

The physical causes of such additively composed mixed distribution are different mechanisms, distributed according to distribution functions $F_1(x)$ in accordance with which the entire process (distributed according to distribution function $F_i(x)$ can take place

$$F(x) = \sum_{i=1}^{j} \alpha_i F_i(x), \text{ where } \sum_{i=1}^{j} \alpha_i = 1 \qquad (1)$$

Multiplicative mixed distributions to be considered here are mathematically very difficult to process, because of the six parameters involved (μ_{Cs}, σ_{Cs} α_{Cs}, μ_{m}, σ_{m}, and α_{m}), which according to (1) and α_{Cs} + + α_{m} = 1 can be reduced to five parameters (μ_{Cs}, σ_{Cs} α_{Cs}, μ_{m}, σ_{Am},). Because of this, mathematical analysis of a mixed distribution consisting of two normal distributions is rather complex and subject to a great uncertainty. There consequently remain empirical methods, which can be employed either using computers, or graphically, using a probability paper. Graphical methods are based on the empirical probability density function $f^*(x)$, which is divided into density functions $f_i^*(x)$ of a particular type of distribution, in such a way that the initial density $f^*(x)$ is obtained as a superposition of $f_i^*(x)$ functions. This procedure is facilitated by the fact that, when normal distribution probability paper is used, the branches of the empirical density function $f^*(x)$ are almost normally distributed if the component densities $f_i^*(x)$ are also normally distributed.

In a graphical approximation, these straight branches should be the starting point. The mixed distribution from fig. 1 is decomposed into its constituent parts in fig. 2.

The characteristic of the mixed distribution F(Cs + Am) and the modes described suggest that F(Cs + Am) is composed of two normal distributions. Since the starting point has to be the empirical density function, the measured values are recorded in a frequency table [12, 13] and processed (tab. 1). The empirical density function is graphically represented (fig. 2) and divided into two "universe" (populations). The relative frequency of each component universe is entered into tab. 1. Frequency sums yield component parameters as α_{Cs} = = 0.42 and α_{Am} = 0.58.

Cumulative frequencies are then formed for each universe, according to

$$h_{\Sigma k} = \frac{1}{\alpha_{cs,Am}} \sum_{i=1}^{k} h_i \qquad (2)$$

(see tab. 1) and represented in order to determine to the parameters graphically as quantiles n_{50Cs}, σ_{50Cs}, and

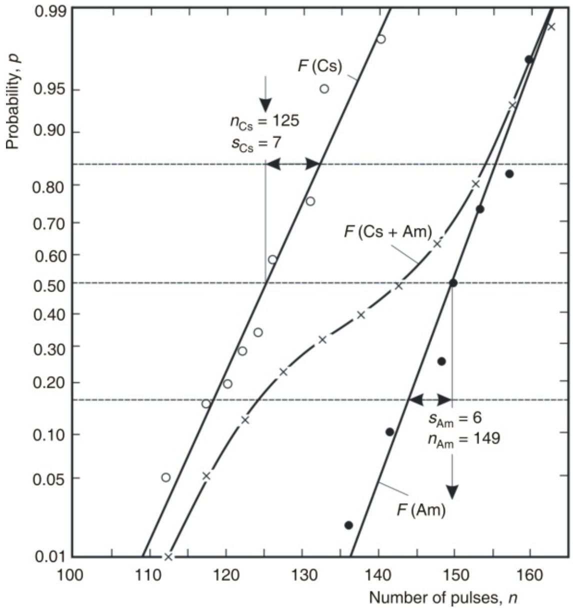

Figure 1. Mixed distribution of the "number of pulses" random variable
F(Cs) – distribution function for the number of pulses from the Cs source; F(Am) – distribution function for the number of pulses from the Am source; F(Cs + Am) – distribution function for the number of pulses from a mixed Cs and Am source

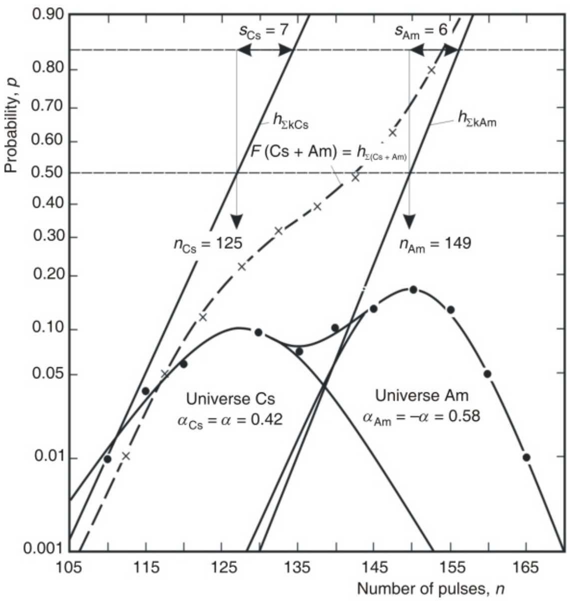

Figure 2. Analysis of mixed distribution *F*(Cs + Am) from fig. 1

n_{50Am}, σ_{50Cs}. The result obtained is a mixed distribution, consisting of two normal distributions (marked with Φ), with the distribution function expressed as

$$F(\text{Cs} + \text{Am}) = 0.42\Phi(n_{\text{Cs}}; 125; 7^2) + 0.58\Phi(n_{\text{Am}}; 149; 6^2) \qquad (3)$$

The parameters of the component distributions are in good agreement with the distribution function for individual sources (figs. 1 and 2).

Table 1. Evaluation of the mixed distribution (fig. 2) with a sample size of *n* = 199

Class number	Class limits		Absolute frequency	Relative cumulative frequency	Relative frequency	Relative frequency Universe Cs	Relative frequency Universe Am	Relative cumulative frequency Universe Cs	Relative cumulative frequency Universe Am
k	n_{Cs}	n_{Am}	h_{mk}	$h_{\Sigma k}$	h_k	h_{kCs}	h_{kAm}	$h_{\Sigma kCs}$	$h_{\Sigma kAm}$
1	>107	112	2	0.010	0.010	0.010	–	0.024	–
2	>112	117	4	0.050	0.040	0.040	–	0.119	–
3	>117	122	12	0.120	0.070	0.070	–	0.286	–
4	>122	127	20	0.220	0.100	0.100	–	0.525	–
5	>127	132	19	0.315	0.095	0.093	0.002	0.747	0.003
6	>132	137	15	0.390	0.075	0.062	0.013	0.895	0.026
7	>137	142	20	0.490	0.100	0.032	0.068	0.971	0.144
8	>142	147	28	0.630	0.140	0.010	0.130	0.995	0.370
9	>147	152	34	0.800	0.170	0.020	0.168	0.999	0.662
10	>152	157	26	0.930	0.130	–	0.130	–	0.888
11	>157	162	11	0.985	0.055	–	0.055	–	0.983
12	>162	167	2	0.995	0.010	–	0.010	–	0.999
			0.995		$\alpha_{Cs} = 0.419$ α 0.42		$\alpha_{Am} = 0.576$ $1 - \alpha$ 0.58		

CONCLUSIONS

The method presented in the paper enables individual radioisotopes in a mixture to be separately detected and characterized. This method also makes possible that a GM counter's dead time be determined according to a single measurement. Because of the mathematical difficulties associated with mixed distributions, they should only be used when that is physically necessitated, *i. e.* when the physical model of the random process being investigated produces a mixed distribution. One should on no account interpret an empirically-found relationship from the outset as a mixed distribution without such a model. Many relationships in nature and technology intrinsically follow mixed distributions, but specific individual influences usually dominate, so that mixed components can be ignored.

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AUTHOR CONTRIBUTIONS

Theoretical analysis was carried out by D. Ć. Dolićanin-Djekić and Dž. F. Pućić. Experiments were carried out by Dž. F. Pućić and K. Dj. Stanković. All of the authors have analysed and discussed the results. The manuscript was written by Dž. F. Pućić. The figures and tables were prepared by K. Dj. Stanković.

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Диана Ћ. ДОЛИЋАНИН-ЂЕКИЋ, Ценис Ф. ПУЋИЋ, Ковиљка Ђ. СТАНКОВИЋ

ПРИМЕНА МЕШОВИТЕ СТАТИСТИЧКЕ РАСПОДЕЛЕ АДИТИВНОГ ТИПА НА АНАЛИЗУ ЗРАЧЕЊА СМЕШЕ РАДИОАКТИВНИХ ИЗОТОПА

У раду се разматра могућност раздвајања ефеката зрачења из два или више радиоактивних изотопа применом метода статистичке математике. Поступак се своди на примену мешовите расподеле адитивног типа. Математички поступак приказан је на примеру анализе сложеног зрачења из два радиоактивна извора.

Кључне речи: зрачење смеше изотопа, мешовита статистичка расподела адитивног типа, параметри појединачних универзума расподеле