KINETIC PARAMETERS STUDY BASED ON BURN-UP FOR IMPROVING THE PERFORMANCE OF RESEARCH REACTOR EQUILIBRIUM CORE

by

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In this study kinetic parameters, effective delayed neutron fraction and prompt neutron generation time have been investigated at different burn-up stages for research reactor's equilibrium core utilizing low enriched uranium high density fuel (U_3Si_2-Al fuel with 4.8 g/cm³ of uranium). Results have been compared with reference operating core of Pakistan research Reactor-1. It was observed that by increasing fuel burn-up, effective delayed neutron fraction is decreased while prompt neutron generation time is increased. However, over all ratio β_{eff}/Λ is decreased with increasing burn-up. Prompt neutron generation time Λ in the understudy core is lower than reference operating core of reactor at all burn-up steps due to hard spectrum. It is observed that β_{eff} is larger in the understudy core than reference operating core of due to smaller size. Calculations were performed with the help of computer codes WIMSD/4 and CITATION.

Key words: delayed neutron fraction, prompt neutron generation time, research reactor, WIMSD/4, CITATION

INTRODUCTION

Kinetic parameters play important role in predicting control and safety performance of any reactor system. Kinetic parameters are a function of core parameters like fuel enrichments, fuel density, neutron spectrum, core size and geometry, moderator type and fuel composition, *etc*. Reactor transient response strongly depends upon the kinetic parameters ratio β_{eff}/Λ [1, 2]. Values of effective delayed neutron fraction and prompt neutron generation time vary with increasing fuel burn-up. Changes in the kinetic parameters depend upon the burn-up limit and must be considered in transient analysis [3, 4]. Fuel density variation results in alteration of fuel to moderator ratio, which causes the change in neutron energy spectrum. By utilizing high density fuel, neutron spectrum becomes hard [5] and in the hard spectrum, fast fission factor is increased. Variation in fast fission factor changes the delayed neutron fraction and average inverse velocity of neutrons which causes a change in the prompt neutron generation time. Effective delayed neutron fraction β_{eff} strongly depends on the core size [6] therefore it must be evaluated if core size is changed. Another factor that affects the kinetic parameters is the positive insertion of reactivity due to which reactor power jumps to a higher level. Due to heating of fuel and coolant, the neutron spectrum becomes hard and variations in kinetic parameters occur for a short time but the effect of this short time is negligible [7].

On improved performance base, equilibrium research reactor core was proposed utilizing low enriched uranium (LEU) high density fuel (U_3Si_2-Al) [8]. The proposed core with high density fuel (uranium density 4.8 g/cm^3) without any changes in the standard and control fuel elements of Pakistan Research Reactor- 1 (PARR-1) comprises 15 standards and 4 control fuel elements. Proposed core is shown in fig. 1 with water boxes at C7 and C4 positions. This is compact and highly under moderated core with ^{235}U enrichment of 19.99% [8]. From three sides it is reflected by graphite blocks while on one side there is graphite thermal column to minimize the neutron leakage and flatting the flux profile. Depletion analysis of core reveals that it can be operated for 41 full power days at 9 MW continuously [8]. Therefore every standard fuel element will be irradiated for 205 full power days while every control fuel element will be for 164 full power days [8].

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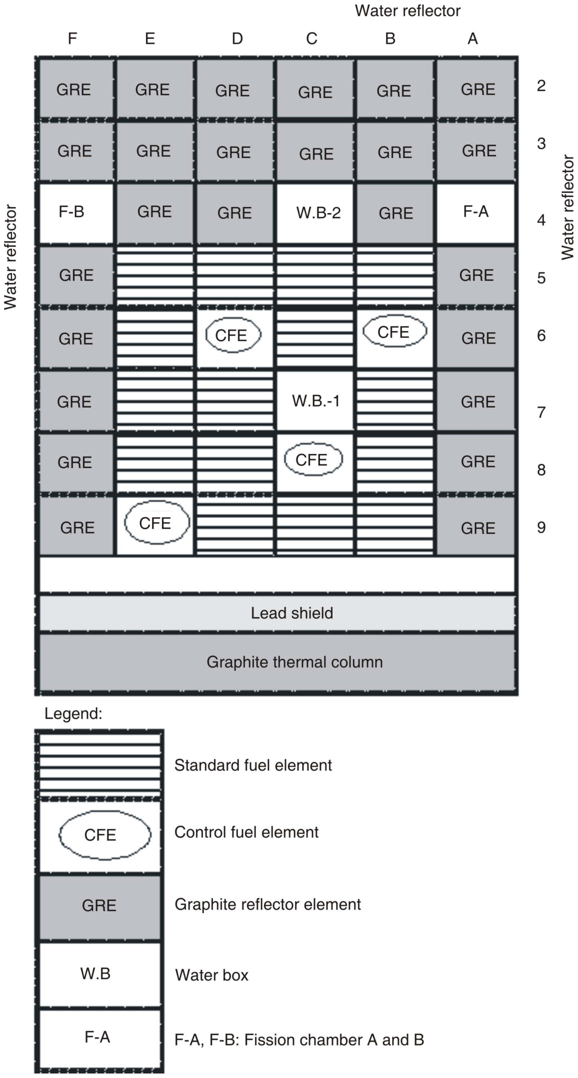

Figure 1. Proposed equilibrium core configuration

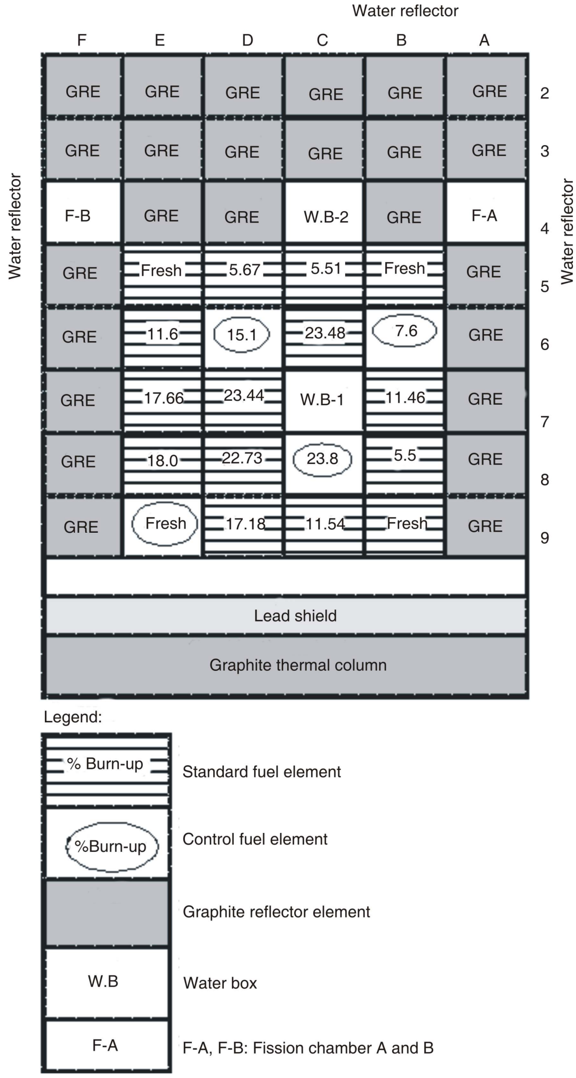

Figure 2. Burn-up (% age of ^{235}U) for proposed core at BOEC

Flow of kinetic parameters in a typical swimming pool type research reactor was investigated in [9] while kinetic parameters of material test reactor with higher density silicide dispersion fuel for a 10 MW IAEA benchmark reactor was studied in [10]. Calculation and measurement of kinetic parameters of PARR-1 reference operating core was studied in [11].

In the current analysis, burn-up dependency of effective delayed neutron fraction and prompt neutron generation time has been studied for the above mentioned equilibrium core [8] for further investigation of the transient analysis.

CALCULATION OF $\beta_{\text{eff}}/\Lambda$

Reactor kinetic parameters (effective delayed neutron fraction and prompt neutron generation time) have been calculated for proposed equilibrium core utilizing higher uranium density of 4.8 g/cm^3. Burn-up (% age of ^{235}U) for proposed core at beginning of equilibrium core (BOEC) and end of equilibrium core (EOEC) is shown in fig. 2 and fig. 3, respectively. Ten energy groups' microscopic cross-sections were obtained through the use of MTR_PC26 package. The energy group structure used is given in tab. 1 [4, 9, 11]. All cross-sections were calculated at 40 °C. Unit cell shown in fig. 4 [12] was selected for generation of cross-sections and number densities of fuel part of standard and control fuel element. Number densities and cross-sections generated by WIMSD/4 [13] and BORGES [14] from MTR_PC26 package were utilized in CITATION [15] for 2-D analyses of the core. Axial buckling of the core was employed in third dimension. All calculations were performed with xenon in equilibrium condition. Average values of thermal and fast flux of the core were evaluated through CITATION. Fast and thermal flux distributions in the core configuration are shown in fig. 5. It is clear from fig. 5 that thermal flux is significantly higher at C4 and C7, the water boxes for sample irradiation. However, among these two locations, maximum thermal neutron flux has been observed at central irradiation facility

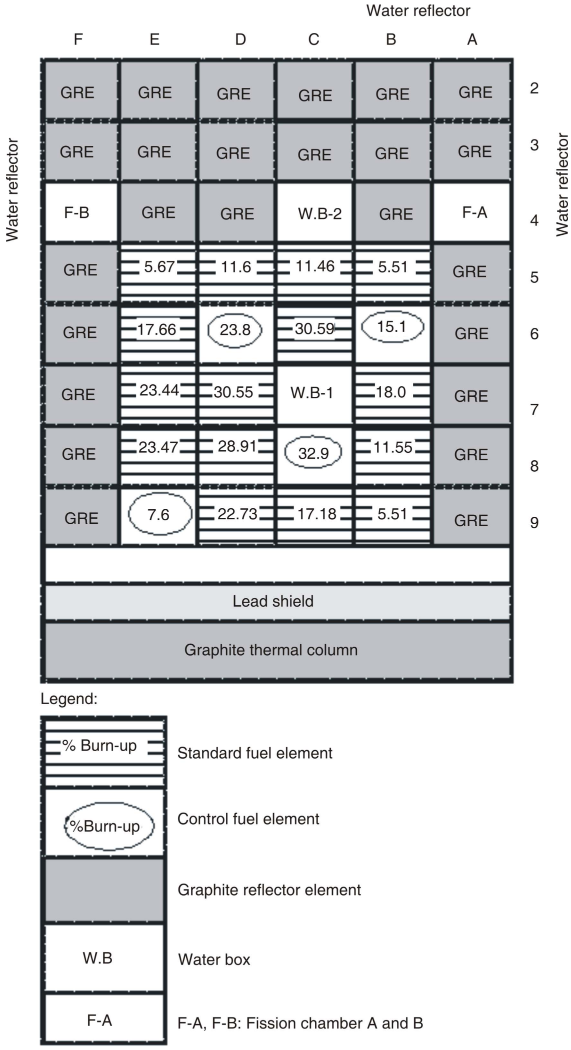

Figure 3. Burn-up (% age of ^{235}U) of proposed core at EOEC

C7. However, fast flux is higher in the central fuel region of the core and decreases due to leakage at peripheries shown in fig. 5. For both thermal and fast fission, values of six group delayed neutron yields ν_{dj}(th) and ν_{dj}(fast) for ^{235}U, ^{238}U, ^{239}Pu, and ^{240}Pu were taken from [16] given in tab. 2. Weighted flux average of these neutron yields for each delayed neutron group of the mentioned isotopes was taken by using the following relation

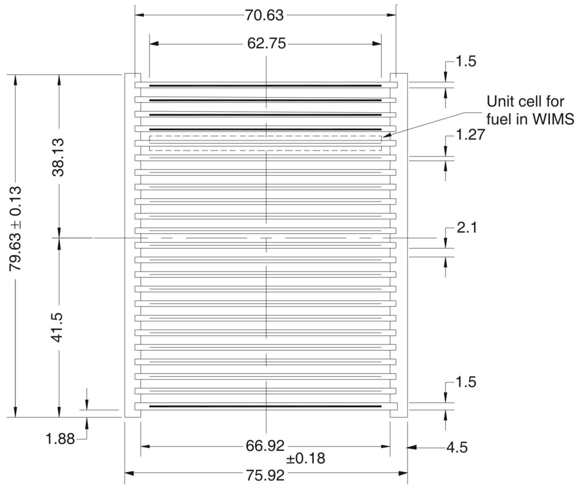

Figure 4. Standard fuel element of PARR-1 with unit cell configuration (all dimensions in mm)

$$\nu_{dj} = \frac{\nu_{dj}(th)\phi_{th} + \nu_{dj}(fast)\phi_{fast}}{\phi_{th} + \phi_{fast}} \tag{1}$$

where ν_{dj} is the total delayed neutron yield of j_{th} group, ν_{dj} (th) – the delayed neutron yield of j_{th} group due to thermal fission, ν_{dj} (fast) – the delayed neutron yield of j_{th} group due to fast fission, ϕ_{th} – the thermal neutron flux and ϕ_{fast} – the fast neutron flux. These flux weighted average values of delayed neutron yields for the above mentioned isotopes were utilized at the input of CITATION. Through the use of perturbation calculations option in CITATION, effective delayed neutron fraction was obtained according to relation

$$\beta_{eff} = \frac{\sum_i V_i \sum_g \chi'(j,g)\phi^*_{i,g} \sum_b \beta_{b,j} N_{b,i} \sum_n \sigma_{f,n,b,i}\phi_{i,n}}{\sum_i V_i \sum_g \chi(g)\phi^*_{i,g} \sum_n \nu\sigma_{f,n}\phi_{i,n}} \tag{2}$$

where V_i represent the volume of region i, $\chi'(j, g)$ – the delayed neutron distribution function, $\beta_{b,j}$ – the de-

Table 1. Ten energy group structure for microscopic cross-sections

Energy group [eV]	Upper energy	WIMSD4 energy group	Average energy [eV]	Remarks
1	$10\ 10^6$	1-5	$2.87\ 10^6$	Above threshold fission of ^{238}U and no delayed production
2	$0.821\ 10^6$	6-7	$4.98\ 10^5$	Average energy of 2 to 6 delayed neutron group produced
3	$3.025\ 10^5$	8	$2.353\ 10^5$	Average produced energy of first delayed neutron group
4	$0.183\ 10^6$	9-20	$8.2\ 10^3$	Fourth and fifth groups are of equal lethargy intervals in WIMSD groups from 9-33
5	367.262	21-33	20.551	
6	1.15	34-40	1.057	Group covers the resonance of ^{240}Pu *i. e.* at 1 eV
7	0.972	41-45	0.78	The group 7 separates the energy boundaries of group 6 and 8
8	0.625	46-55	0.296	This group covers the resonance of ^{239}Pu *i. e.* at 0.3 eV
9	0.14	56-60	0.0837	Ninth group is based on the maximum thermal neutron cut-off energy, 0.14 eV (5 kT)
10	0.050	61-69	$2.2\ 10^{-4}$	Remaining thermal groups

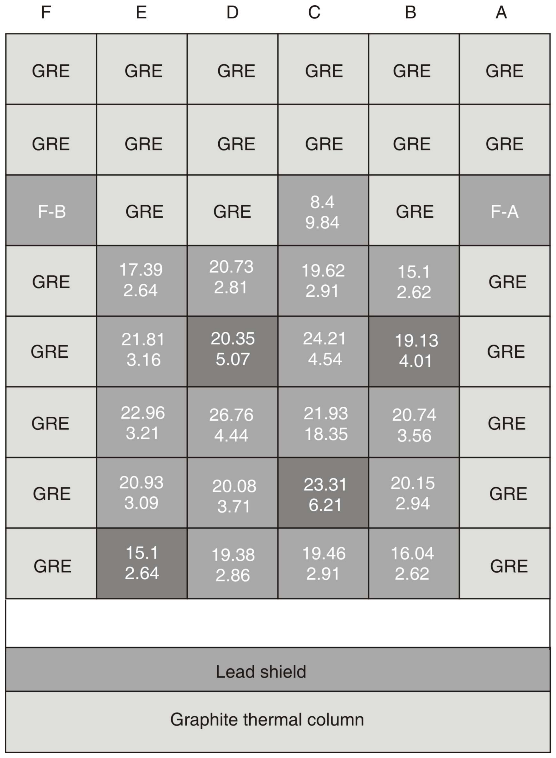

Figure 5. Fast (upper value) and thermal (lower values) neutron flux (10^{13} $cm^{-2}s^{-1}$) in the proposed core

layed neutron fraction of delayed family j at location i for nuclide b and $N_{b,i}$ – the number density of nuclide b at location i, ϕ – the neutron flux, ϕ^* – the adjoint flux, χ – the distribution function, and ν – the average number of neutron per fission. For prompt neutron generation time, the CITATION code calculates the prompt neutron life time (l) and multiplication factor (k) can then be calculated by relation $\Lambda = l/k$ s. For estimation of prompt neutron lifetime through CITATION, the code requires as its input the flux weighted, average inverse velocity, for each energy group according to relation

$$\overline{\frac{1}{\upsilon}} = \frac{\int_0^\infty \frac{1}{\upsilon(E)}\phi(E)\mathrm{d}E}{\int_0^\infty \phi(E)\mathrm{d}E} \tag{3}$$

where $\upsilon(E)$ is the energy dependent neutron velocity. The value of $\overline{(1/\upsilon)}$ is calculated by introducing small amount of $(1/\upsilon)$ absorber in different regions of core [4]. The spectrum averaged cross-section for $(1/\upsilon)$ absorber is calculated by the relation [17].

$$\overline{\sigma_a(E)} = \frac{\int_0^\infty \frac{\sigma_{a,0} v_0}{\upsilon(E)}\phi(E)\mathrm{d}E}{\int_0^4 \phi(E)\mathrm{d}E} \tag{4}$$

$$\overline{\frac{1}{\upsilon}} = \frac{\overline{\sigma_a(E)}}{\sigma_{a0} v_0} = \frac{\int_0^\infty \frac{1}{\upsilon(E)}\phi(E)\mathrm{d}E}{\int_0^\infty \phi(E)\mathrm{d}E} \tag{5}$$

where $\sigma_{a,0} = 1.0$ b (1 b $= 10^{-28}$ m^2) in WIMSD/4 [18] library at most probable velocity *i. e.* υ_0 = 2200 m/s at most probable energy, *i. e.* 0.0253 eV.

By introducing $(1/\upsilon)$ absorber in WIMSD/4 input and keeping the neutron velocity at most probable energy υ_0 = 2200 m/s in BORGES input, BORGES calculates the average inverse velocity of neutron for each group and writes it in the microscopic cross-section output file in the CITATION input format. Utilizing the average inverse velocities of neutron, the CITATION code computes the prompt neutron life time by the relation

$$l = \frac{\sum_i \frac{V}{\upsilon(n)}\phi^*_{i,n}\phi_{i,n}}{\frac{1}{k}\sum_i V_i \sum_g \chi(g)\phi^*_{i,g} \sum_n \nu\Sigma_{f,n}\phi_{i,n}} \tag{6}$$

$\upsilon(n)$ is the neutron velocity and Σ_f is the macroscopic fission cross-section.

RESULTS AND DISCUSSIONS

Effective delayed neutron fractions and prompt neutron generation times were calculated at different steps of burn-up cycle for the understudy proposed core. Table 3 indicates that by increasing core burn-up, effective delayed neutron fraction is decreasing while prompt neutron generation time is increasing. With increasing burn-up of fuel, initial loading of ^{235}U is depleted; ^{239}Pu and its higher isotopes are produced due to internal conversion of ^{238}U. Plutonium isotopes are

Table 2. Group based delayed neutron yield of different isotopes for fast and thermal flux

Isotopes	Yield	Group-1	Group-2	Group-3	Group-4	Group-5	Group-6
^{235}U	v_{dj}(fast)	0.000630	0.003510	0.003100	0.006720	0.002110	0.000430
	v_{dj}(th)	0.000520	0.003460	0.003100	0.006240	0.001820	0.000660
^{239}Pu	v_{dj}(fast)	0.000240	0.001760	0.001360	0.002070	0.000650	0.000220
	v_{dj}(th)	0.000210	0.001820	0.001290	0.001990	0.000520	0.000270
^{238}U	v_{dj}(fast)	0.000540	0.005640	0.006670	0.015900	0.009270	0.003090
^{240}Pu	v_{dj}(fast)	0.000280	0.002380	0.001620	0.003150	0.001190	0.000240

Table 3. Burn-up dependent effective delayed neutron fraction and prompt neutron generation time

	Reference operating ore of PARR-1		Proposed core of PARR-1	
	Effective delayed neutron fraction β_{eff}	Prompt generation time Λ [s]	Effective delayed neutron fraction β_{eff}	Prompt generation time Λ[s]
BOC	0.00730**	4.276 10^{-5**}	0.00753*	3.2517 10^{-5*}
14 full power days burn-up	0.00728**	4.3609 10^{-5**}	0.0075*	3.3256 10^{-5*}
35.26 full power days burn-up	0.00725***	4.5032 10^{-5***}	0.00748*	3.4252 10^{-5*}

*present study, **Iqbal *et al*., 2008 [8], ***Muhammad *et al*., 2011 [11]

contributing in power generation as burn-up increases and neutron flux spectrum is also changed. The delayed neutron energy spectra $\chi_{\text{dj}}(E)$ depend upon the isotope from which the delayed neutrons are emitted. Due to plutonium contribution in power generation, the delayed neutron yield and energy spectra $\chi_{\text{dj}}(E)$ is changed. Therefore with increasing burn-up β_{eff} is changed and prompt neutron generation time is mainly changing with changes in average inverse velocities of the neutron in different regions of the core. The effective delayed neutron fraction is decreasing while prompt neutron generation time is increasing with burn-up; therefore reactor response to transient is also changing with increasing burn-up. The spatial and energy distortion in the neutron flux during burn-up cycle changes the β_{eff} and Λ.

Higher fuel density of proposed core results in higher fuel to moderator ratio in proposed core as compared to the reference operating core of PARR-1. Higher fuel to moderator ratio in proposed core causes neutron energy spectrum to be harder [8]. Low average inverse velocities in harder neutron energy spectrum of the proposed core place prompt neutron generation time on lower side. Therefore, prompt neutron generation time in the proposed core is lower than reference operating core of PARR-1 at all burn-up steps (tab. 3). However, due to smaller size of reference core, increase in fast non leakage probability dominates the decrease in fast fission factor by delayed neutrons and importance of delayed neutrons is increased. In general, proportion of $\beta_{\text{eff}}/\Lambda$ is decreasing with increasing burn-up for both analyzed cores as shown in fig. 6.

CONCLUSION

Prompt neutron generation time is low, while effective delayed neutron fraction is high for the smaller sized higher density proposed core as compared to the larger sized lower density reference operating core of PARR-1 by keeping enrichment to be conserved in both reactor cores. Overall ratio ($\beta_{\text{eff}}/\Lambda$) of kinetic parameters is decreasing with increasing burn-up for proposed core of PARR-1.

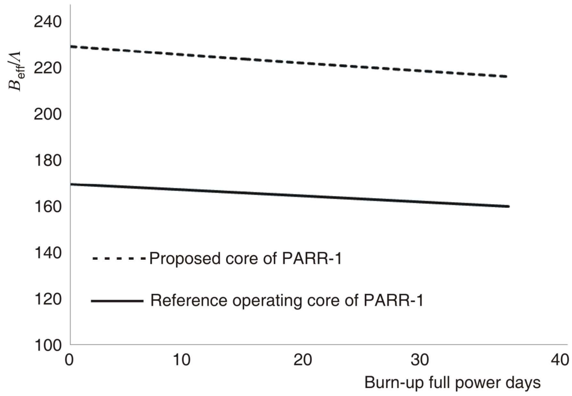

Figure 6. Effective delayed neutron fraction over prompt neutron generation time $\beta_{\text{eff}}/\Lambda$ as a function of burn-up

AUTHOR CONTRIBUTIONS

Calculations were carried out by A. Muhammad and T. Mahmood under supervision and guidelines of M. Iqbal. All authors discussed the result. Manuscript was written by A. Muhammad and reviewed by T. Mahmood. Figures were prepared by A. Muhammad. M. Iqbal checked the manuscript and added corrections.

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ПРОУЧАВАЊЕ КИНЕТИЧКИХ ПАРАМЕТАРА СА ИЗГАРАЊЕМ РАДИ ПОБОЉШАЊА СВОЈСТАВА РАВНОТЕЖНОГ ЈЕЗГРА ИСТРАЖИВАЧКОГ РЕАКТОРА

У овом раду приказано је истраживање кинетичких параметара, ефективне фракције закаснелих неутрона и времена настанка промптних неутрона, за различите нивое изгарања, у равнотежном језгру истраживачког реактора са нискообогаћеним уранијумским горивом високе густине (U_3Si_2-Al гориво са 4.8 g/cm^3 уранијума). Резултати су упоређени са референтним вредностима радног језгра Пакистанског истраживачког реактора-1. Запажено је да се са порастом изгарања горива смањује ефективна фракција закаснелих неутрона, док време настанка промптних неутрона расте. Отуда се укупан однос β_{eff}/Λ смањује са порастом изгарања. Услед тврдог спектра, у свим корацима изгарања време настанка промптних неутрона Λ у проучаваном језгру краће је од онога у референтном радном језгру. Запажено је да је β_{eff} веће у проучаваном језгру него у референтном радном језгру услед мање величине. Прорачуни су обављени програмима WIMSD/4 и CITATION.

Кључне речи: фракција закаснелих неутрона, време настанка промптних неутрона, истраживачки реактор, WIMSD/4, CITATION