ANALYTICAL COMPUTATION TECHNIQUE FOR CALCULATION THE EFFECTIVE GEOMETRICAL SOLID ANGLE AND THE EFFICIENCY OF CUBIC SCINTILLATION CRYSTAL WITH SIDE CYLINDRICAL HOLE

by

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In the gamma-ray spectroscopy field, the radiometric examination for small quantities of natural samples is extremely significant. Therefore, the gamma-ray spectrometry calibration process must be prepared with good precision for several energies, matrices of sources or samples, and source-to-detector shapes. This manuscript considers a new uncomplicated analytical computation technique to calculate the effective geometrical solid angle and the efficiency of cubic scintillation crystal with a side cylindrical hole. The computations can be done by using a simple method, with a few essential limitations, that describes radioactive point sources located inside the side cylindrical hole and a high-efficiency cubic NaI(Tl) detector, come together with a low background as well. The technique stands on a trouble-free solid angle analytical formula for the detection system, using an accurate relation for the detector cavity, united with rough formulas controlling the interactions in the gamma-ray source and the materials introduced in between the source and the gamma-ray spectrometry. This new technique is not restricted to certain sources, because several source shapes can correspond to a homogeneous huge number of point sources and the detector geometry can be represented as a set of border points. The technique simply can be useful to obtain the full-energy peak efficiency in the future, challenging developments for low-energy gamma-ray spectroscopy.

Key words: analytical computation technique, cubic scintillation crystal with side cylindrical hole, geometrical solid angle, detector efficiency

INTRODUCTION

One of the most complex instruments in the radiation measurement field and nuclear examinations is the gamma-ray spectrometer, the power of the numerical values supplied by it are based on the accuracy of the calibration process for these detectors. The gamma-ray photons can travel within the materials by creating very slight interactions, hence this phenomenon leads to lower efficiency of the detectors in comparison with the other types of radiation. Therefore, precise information on gamma-ray spectrometer efficiency has greater weight [1, 2]. In general, the detection system efficiency can be based strongly on several features such as the detector and the source dimensions, the energy range, the arrangement between the source and the detector, attenuation of surrounding layers, besides the density and the composition of the source material itself. Therefore, not all the sophisticated calibration set-ups for some detector can be valid for others. The experimental routine calibration process can be a more authoritative method for the majority of the detectors under their normal conditions, but these detectors have to be recalibrated again for each sample configuration different from the used standard calibration source structure. Several studies were done on the calibration process for different gamma-ray detectors with different geometrical radioactive sources [2-8].

The scintillator detectors are used for gamma-ray photon detection, due to the high detection efficiency which is a significant prerequisite in gamma-ray spectrometry for high-quality pulse-to-noise relative ratio, where the pulse-to-noise relative ratio increases as a function of the detection system efficiency [9, 10]. The perfect scintillator type must have more radiation solidity, high-quality energy response

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and resolution time, more light yield, perspicuous to visible light, superior steadiness in light production with modifying the medium temperature, large Z number, and high material density [11], where the gamma-ray spectroscopy applications need such detector characteristics. The detector efficiency is determined as a registered fraction of the radiation, detected by the detector medium, from the total amount of radiation produced by the source. Therefore, the researchers are always attracted to looking for materials with high total and full-energy peak efficiency, or computation techniques for the detection system, especially in case of low activity measurements [12-24].

The well-type detectors, in general, are very useful in the case of samples that release very low energy gamma-ray photons from certain radionuclides, where there is a huge weight for the self attenuation inside the sample material [24, 25]. The well-type detectors are used as an alternative coaxial detector type with the equivalent volume, due to the detection solid angle in order of 4 , which is higher in comparison with the solid angle of the standard coaxial detector types [25, 26]. The well-type detectors are ideal in case of high material density, low energy gamma-ray photons, high Z, and high gamma-ray photon attenuation inside the sample, the detection efficiency can sometimes reach up to 4 or 5 times higher value than the efficiency of typical coaxial type detector with the same volume [27, 28]. One of the main reasons to use the well-type detector is offered by little quantities of materials that need to be analyzed, which are introduced inside the detector well cavity, to increase the detector efficiency and drop the effect of the self-absorption inside the sample itself [25].

The current manuscript aims to explain a new specific analytical computation technique ACT, depending on the detection probability of the emitted gamma-ray photons from the source, to calculate the effective geometrical solid angle and the total efficiency of the cubic scintillation crystal with a side cylindrical hole. This new type of detector is considered a new ideal and especially well-type instrument for fuel rod scanning applications and for the radioimmunoassay method, where the sample, or the source, can be sited inside the side cylindrical hole, that is set at a 90-degree angle to the cubic scintillation NaI(Tl) detector axis. The production of light from the detector crystal can be accumulated by an optical window, connected to the detector surface from one side. The background level is supposed to be decreased inside the side cylindrical hole in the well-type detectors, and the detector efficiency will be increased, which will help to overcome the detection limitations for short and extensive energy range, where the average -ray photons path length through the detector medium is small in case of high -ray photons. The detector efficiency can be given and studied as a function of the position of the source through the side cylindrical hole and the gamma-ray photon energy.

The ACT is developed to estimate the solid angle for certain geometries of a radioactive point source positioned in the side cylindrical hole of the cubic scintillation crystal detector geometry, by using the formulas of simple integrals, which can be solved numerically by a written easy computer program using BASIC language. This technique can be helpful in the gamma-ray spectrometry system, especially for good accuracy of the calibration process, but firstly, the geometrical solid angle for axial and non-axial point source inside the detector cavity must be studied and determined as the starting step for this technique, then the technique can be expanded to cover the sources in form of disk or volume within the cavity. The gamma-ray photon's path lengths through the source-to-cubic detector system and the attenuation effect, due to all the different factors that exist in the detection system, can be included in the efficiency calculations in that technique, which can be adapted in the future to compute the efficiency of any new gamma-ray spectrometry detectors inside or outside the laboratory. The details of the ACT are presented in the section of the mathematical model after the introduction section. The details of the calculation results and discussion beside the conclusion are given in the sections of results and discussion plus the conclusion.

MATHEMATICAL MODEL

In the current research, the ACT to calculate the effective geometrical solid angle and the efficiency of a cubic scintillation crystal with a side cylindrical hole, was established. This detector type was developed especially with an extraordinary hole in the center, to use in the detection process, for low intensity and low energy of X– and gamma-ray photons, in a lower energy ambient background during the measurements, which was a big challenge for the scientists and technical persons who build up a new design for the gamma-ray detection system and its setup. The model of the ACT must contain accurate geometrical details of the source-to-detector system and needs the gamma-ray laboratory users to know every small factor of an effect on the detection system arrangement, which has to be included in the model. The cylindrical detector under study has a length (2a), width (2b), and height (2c) beside radius, *r*, and length (2b) for the center cylinder hole. The detector was irradiated from inside the side cylindrical hole by using a radioactive point source, positioned on the main axis of the hole and moved on to be at certain axial and non-axial positions with lateral displacement, ρ, as shown in fig. 1. The detector geometry gives the ability to cover approximately 4 solid angles around the source.

The geometrical efficiency $\varepsilon_{G(Hole)}$ the chance that -ray photons pass within the detector in the case of both axial and non-axial point sources inside the detector hole can be given by

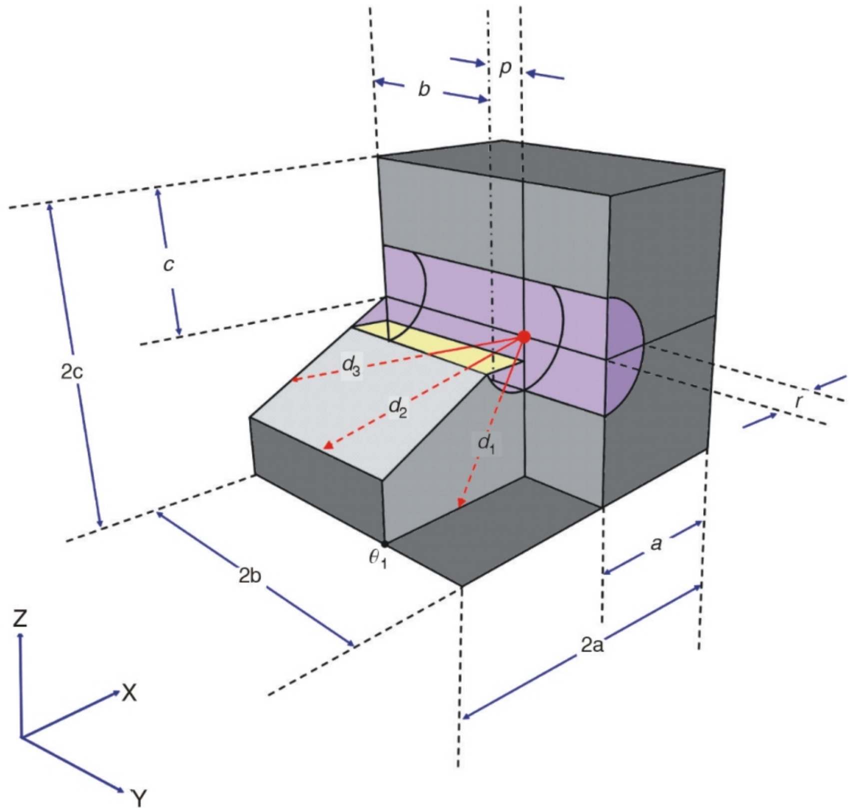

Figure 1. A radioactive point source is located at a distance ρ from the detector major axis, and the probable path lengths are d_1, d_2, and d_3

$$\varepsilon_{\mathrm{G(Hole)}} = \frac{1}{4\pi} \int_{\mathrm{G(Bore)}} \mathrm{d}\Omega_{\mathrm{G(Hole)}}, \mathrm{d}\Omega_{\mathrm{G(Hole)}} = \sin\theta \mathrm{d}\theta \mathrm{d}\theta \mathrm{d}\varphi \tag{1}$$

where the source-to-detector pure solid angle, $\Omega_{\mathrm{G(Hole)}}$, can be defined as the fraction of a certain angular range based on both polar θ and azimuthal φ angles in 3-D dimensions space and given by

$$\Omega_{\mathrm{G(Hole)}} = \int_{\varphi}\int_{\theta} \sin\theta \mathrm{d}\theta \mathrm{d}\varphi \tag{2}$$

where, the polar angle θ obtains values established from 0 up to π/2, while the azimuthal angle φ obtains values established from 0 up to φ_1 and 0 up to φ_2.
By taking into consideration all the photon's path lengths within the detector and border materials, the photon attenuation inside the detector itself, and the other surrounding materials, f_{att}. The effective solid angle $\Omega_{\mathrm{Eff(Point-Hole)}}$ for a non-axial point source placed inside the detector hole can be calculated by

$$\Omega_{\mathrm{Eff(Point-Hole)}} = 4[I_1 + I_2] \tag{3}$$

where I_1 and I_2 can be described as

$$I_1 = \int_0^{\frac{\pi}{2}} \int_0^{\varphi_1} f_{\mathrm{att}} f_i \sin\theta \mathrm{d}\varphi \, \mathrm{d}\theta \quad \text{and}$$

$$I_2 = \int_0^{\frac{\pi}{2}} \int_0^{\varphi_2} f_{\mathrm{att}} f_i \sin\theta \mathrm{d}\varphi \, \mathrm{d}\theta \tag{4}$$

where $f_i = (1 - \mathrm{e}^{-\mu d_i})$, for $i = 1$, 2, and 3

The chance that γ-ray photons traveling a distance d_i in the detector medium with no interaction is $e^{-\mu di}$, so its interaction chance is $f_i = (1 - e^{-\mu d_i})$, where μ is the total macroscopic cross-section of the detector's material. The values of φ_1 and φ_2 are plotted in fig. 2 and they can be represented by the following eq.

$$\varphi_1 = \tan^{-1}(m_1/x) \text{ and } \varphi_2 = \tan^{-1}(m_2/x) \tag{5}$$

where $m_1 = b - \rho$ in case of I_1 and $m_2 = b + \rho$ in case of I_2.

In addition, d_i, symbolizes the possible photon path lengths through the detector material itself and can be defined as, d_1, d_2, ands d_3, respectively, as shown in fig. 2. If the photons exit from XY, YZ, or XZ planes respectively, the previous three path lengths can be established using the following expressions

$$d_1 = \frac{c}{\cos(\theta)} - \frac{x}{\sin(\theta)\cos(\theta)}$$
$$d_2 = \frac{a - x}{\sin(\theta)\cos(\theta)} \tag{6}$$
$$d_3 = \frac{1}{\sin(\theta)}\left(\frac{m}{\sin(\varphi)} - \frac{x}{\cos(\varphi)}\right)$$

where $m = m_1$ or m_2 for I_1 and I_2 respectively.

The polar angles $\theta_{\varphi 1}$ and $\theta_{\varphi 2}$ are correlated with the azimuthal angles φ_1 and φ_2, respectively, and it can be expressed by the following equation

$$\theta_{\varphi 1} = \tan^{-1}\sqrt{x^2 + (b - \rho)^2}/z$$
$$\theta_{\varphi 2} = \tan^{-1}\sqrt{x^2 + (b + \rho)^2}/z \tag{7}$$

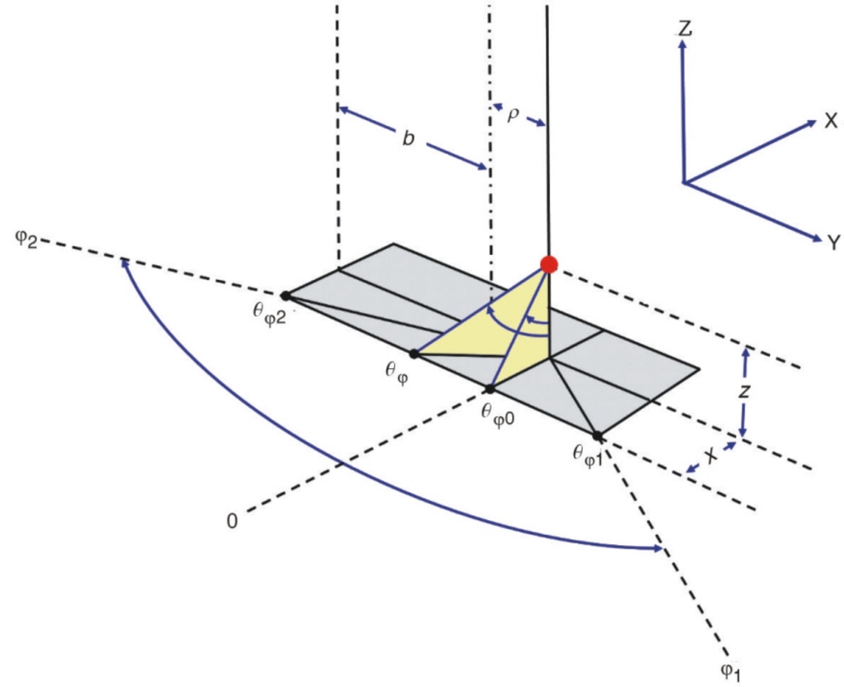

Figure 2. A horizontal cross-sectional view of the detector side hole

while the polar angles $\theta_{\varphi0}$ and θ_{φ} are correlated with the azimuthal angles 0 and φ, respectively, and this can be determined by the following equation

$$\theta_{\varphi0} = \tan^{-1}(x/z) \\ \theta_{\varphi} = \tan^{-1}[x/z\cos(\varphi)] \tag{8}$$

where x and z are represented in fig. 2 and can be calculated as functions of θ_{0} by the following equation

$$x = r\sin(\theta_{\varphi0}), z = x/\tan(\theta_{\varphi0}) \tag{9}$$

The polar and azimuthal angles that govern the incident photon's pathways through the detector material can be seen in fig. 3. It is found that θ_1, represents the maximum polar angle for the photons to leave the detector active medium from its base at certain values $\varphi = 0$, where θ_1 can be calculated using eq. (10). Moreover, if the value of the azimuthal angle $\varphi > \varphi_s$ the incident photon will leave the detector by side of the XZ-plane

$$\theta_1 = \tan^{-1}(a/c) \tag{10}$$

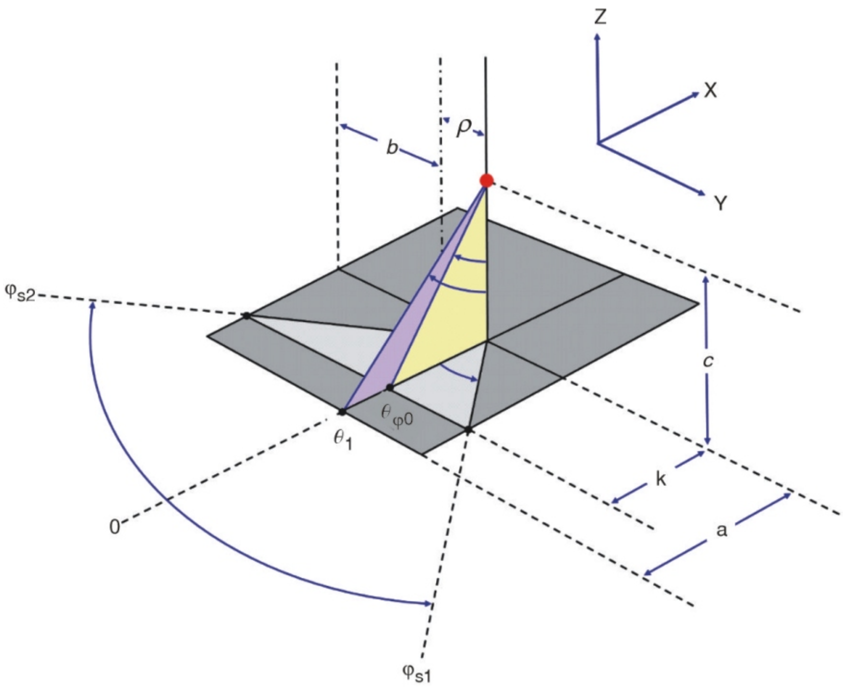

Figure 3. Polar and azimuthal angles for a radioactive point source concerning the cubic detector base (XY-plane)

The azimuthal angle φ_s takes two different values φ_{s1} and φ_{s2}, as shown in fig. 3; both φ_{s1} and φ_{s2} can be calculated by

$$\varphi_s \quad \varphi_{s1} \quad \tan^{\quad 1} \frac{b \quad \rho}{k}$$

$$\varphi_s \quad \varphi_{s2} \quad \tan^{\quad 1} \frac{b \quad \rho}{k} \tag{11}$$

where

$$k \quad c \tan(\theta_{\varphi 0}) \tag{12}$$

The values of the path lengths, in general, depend directly on the values of the polar and the azimuthal angles under certain conditions which are listed

$$\begin{aligned} &d \quad d_1 \text{ if } (\theta \quad \theta_1 \text{ and } \varphi \quad \varphi_s) \\ &d \quad d_2 \text{ if } (\theta \quad \theta_1 \text{ and } \varphi \quad \varphi_s) \\ &d \quad d_3 \text{ if } (\varphi \quad \varphi_s) \end{aligned} \tag{13}$$

The average path length value $\bar{d}_{(\mathrm{Point} \quad \mathrm{Hole})}$ in the cubic scintillation crystal, with a side cylindrical hole, based on the position of the incoming photons from the source, can be calculated according to

$$\bar{d}_{(\mathrm{Point} \quad \mathrm{Hole})} \quad \frac{\displaystyle \int_G \sum_{i \quad 1}^{n} d_i(\theta,\alpha) \, \mathrm{d} \quad G}{\displaystyle \int_G \mathrm{d} \quad G} \quad \frac{\displaystyle \int_\varphi \int_\theta \sum_{i \quad 1}^{n} d_i(\theta,\alpha) \, \sin\theta \mathrm{d}\theta \mathrm{d}\varphi}{G} \tag{14}$$

$$\bar{d}_{(\mathrm{Point} \quad \mathrm{Hole})} \quad \frac{1}{2} \left[\frac{\displaystyle \int_0^{\frac{\pi}{2}} \int_0^{\varphi_1} d_i(\theta,\varphi) \sin\theta \mathrm{d}\varphi \mathrm{d}\theta}{\displaystyle \int_0^{\frac{\pi}{2}} \int_0^{\varphi_1} \sin\theta \mathrm{d}\varphi \mathrm{d}\theta} \quad \frac{\displaystyle \int_0^{\frac{\pi}{2}} \int_0^{\varphi_2} d_i(\theta,\varphi) \sin\theta \mathrm{d}\varphi \mathrm{d}\theta}{\displaystyle \int_0^{\frac{\pi}{2}} \int_0^{\varphi_2} \sin\theta \mathrm{d}\varphi \mathrm{d}\theta} \right] \tag{15}$$

The total efficiency $\varepsilon_{T(\mathrm{Point} \quad \mathrm{Hole})}$ of the cubic scintillation crystal with a side cylindrical hole, in the case of non-axial point sources located inside the detector hole, can be defined as the ratio between the number of gamma-ray photons that are recorded in the detector, with any possible energy, during a certain time interval, and the number of gamma-ray photons that are emitted by the source during the same time interval [29]. It is related to the intrinsic total efficiency ε_{iT} by the following equation

$$\varepsilon_{\mathrm{T(Point \quad Hole)}} \quad \varepsilon_{\mathrm{G(Hole)}} \varepsilon_{iT} \quad \frac{\mathrm{eff(Point \quad Hole)}}{4} \tag{16}$$

where $\varepsilon_{iT} \quad f_{att} f_i \quad f_{att}(1 \quad \mathrm{e}^{\quad \mu d_i})$

Usually, not all gamma-ray photons that enter the detector will be recorded with it, depending on the detector dimensions, the photon energy, and the detector material type. Therefore, intrinsic total efficiency ε_{iT} can be defined as the ratio between the number of photons that are recorded by the detector and the number of photons that enter the detector [29]. Moreover, it can represent the chance that gamma-ray photons will go through an interaction inside the detector material. Now the equation number (16) can be rewritten in detail by using

$$\varepsilon_{T(\mathrm{Point} \quad \mathrm{Hole})} \quad \frac{1}{\quad} \int_0^{\frac{\pi}{2}} \left[\int_0^{\varphi_1} f_{att} f_i \sin\theta \mathrm{d}\varphi \quad \int_0^{\varphi_2} f_{att} f_i \sin\theta \mathrm{d}\varphi \right] \mathrm{d}\theta \tag{17}$$

By using the definition of the average path length value $\bar{d}_{(\mathrm{Point} \quad \mathrm{Hole})}$ in the cubic scintillation crystal with a side cylindrical hole, in eq. (15), the intrinsic total efficiency, ε_{iT}, can be calculated by the following relation

$$\varepsilon_{iT} \quad f_{att} f_i \quad f_{att}[1 \quad \mathrm{e}^{\quad \mu \bar{d}_{(\mathrm{Point} \quad \mathrm{Hole})}}] \tag{18}$$

Numerical evaluation of all the integrations was executed using the trapezoidal rule, where a computer program was prepared concerning the source at any separation axial and non-axial source-to-detector distance inside the hole, while the precision of the integration came together very well at the number of the intervals n under every integration reach to $n = 20$. By setting the value of the lateral distance ρ equal to zero, the axial value of any physical quantities that are defined at the top, can be calculated simply.

RESULTS AND DISCUSSION

Based on the scintillation detector's advantages, where it has high efficiency and can be fabricated in different sizes and shapes, the NaI(Tl) has many applications in medical and industrial zones. The novel capability of the cubic scintillation crystal with a side cylindrical hole exhibits a high challenge in comparison to the elderly shapes that were prepared by several manufacturers, in the cylindrical form [30]. The detection process can be done when the gamma-ray photons strike the detector from the square side, which is called the entrance window of the detector, or when the gamma-ray photons strike the detector through the side cylindrical hole of the detector. Typically, the side-well detectors are used in applications that would not allow for the height of a straight-on detector design and require special detector geometries. The side-well detector design is hard to be manufactured, since the scintillation light must travel around the well, it is very difficult to compensate for the intrinsic losses. More recently, many applications that would have used a side-well detector are solved with shorter photomultiplier tubes, or silicon photomultipliers. Moreover, there is an alternative design that does not exhibit the losses that are inherent in the detector medium, such as the detector under the present study, where the crystal is such that interactions taking place in different parts of the scintillation material give the same signals in the PMT [30].

It was motivating and useful to perform a broad study on a new uncomplicated ACT to reach accurate advantages of the cubic NaI(Tl) scintillation crystal with a side cylindrical hole within 0.05 up to 3.00 MeV as the energy range. This work is most important for the knowledge about the detection process behavior when the point source is located within the side cylindrical hole of the cubic detector at different positions. The most important point, in this case, is to obtain the true solid angle exactly and the gamma-ray photon path length within a detector material. In the high-energy region, determining the efficiency of such a detector is too complicated because there is no standard radioactive calibration source that releases gamma-ray photons with energy in the region of 3 MeV. Consequently, the experimental calibration of such a detector and the efficiency calculations with high energy can not be done by using the standard -ray sources in practice. Besides all of that, it will not succeed to make interpolation and extrapolation for a few measured efficiency points, to cover all the regions with the best fitting function, especially in the high energy region. To rise above these troubles, the ACT was introduced in this paper to calculate the geometrical solid angle $_{\mathrm{G(Hole)}}$, the effective solid angle $_{\mathrm{Eff(Point\ \ Hole)}}$, the geometrical efficiency $\varepsilon_{\mathrm{G(Hole)}}$, and the total efficiency $\varepsilon_{T\mathrm{(Point\ \ Hole)}}$, of cubic NaI(Tl) scintillation crystal with a side cylindrical hole, at different locations and energies, inside the detector hole.

The gamma-ray photon average path length $\bar{d}_{\mathrm{(Point\ \ Hole)}}$ within the detector material, was calculated as well, therefore mathematically the detector can be calibrated, and obtaining values of the efficiency with high accuracy may be the study's need for fine-tuning with the available measured data at several points of energy. The geometrical drawing for cubic NaI(Tl) scintillation crystal with a side cylindrical hole was supplied by the SCIONIX HOLLAND BV company for scintillation detectors and radiation detection instruments [31]. The type of detector was V76AP76/3M-X, while the geometrical number for the drawing was VS-1104-40, which contains the PM tube and its connections with the detector, including all the specifications, as shown in fig. 4. The XCOM program [32] was used to calculate the total macroscopic cross-section of the detector's material, the detector window material, and the rest of the layers that exist in the detector structure. All the previous calculations were done by assuming the radioactive source placed on the hole major Y-axis, has a point-like shape, as shown in fig. 5, at eight different lateral distances ρ which start from 0 cm up to 3.5 cm. The obtained results were compared with the published data for the cylindrical NaI (Tl) detector with a full side hole [30], under the same conditions of energy, source shape, and source position. To obtain the effective solid angle, $_{\mathrm{Eff(Point\ \ Hole)}}$ accurately, the photon's path length through the detector active medium was estimated.

The geometrical solid angle $_{\mathrm{G(Hole)}}$ and the geometrical efficiency $\varepsilon_{\mathrm{G(Hole)}}$ were calculated at each lateral distance ρ inside the cubic detector hole and it was compared with the cylindrical detector results as reported in tab. 1. It was found that the values of both have the same behavior, where they were decreased as the lateral distance ρ increased. In addition, it was noticed that the values of both of them, in the case of the cubic detector, were slightly greater than those for the cylindrical detector, this is due to an insignificant difference in the volume, where the volume of the cylinder detector is 304 cm^3 and while the volume of the cubic detector is 397 cm^3, while both have the same inner radius 1.32 cm and the same hole length 7.6 cm.

The effective solid angle of the cubic detector was calculated for axial and non-axial positions to show the variation of lateral distance ρ on its value. This study was done by using a radioactive point source located on the detector hole major Y-axis, at eight positions. The change of the effective solid angle $_{\mathrm{Eff(Point\ \ Hole)}}$ for the cubic detector, as a function of photon energy, was plotted in fig. 6 and compared with the values of the cylindrical detector [30]. The graphs show that the effective solid angle $_{\mathrm{Eff(Point\ \ Hole)}}$ was decreased as the lateral distance, ρ, increased for both cubic and cylindrical detectors. Moreover, there was an intersection point between the curves for both detectors, this intersection point moved to the low energy region as the lateral distance, ρ, increased. At all the values of the lateral distance ρ, the value of effective solid angle $_{\mathrm{Eff(Point\ \ Hole)}}$, for the cubic detector in the region of low energy, is the higher one compared to the cylindrical detector values due to the photoelectric phenomena, being governed by the interactions with a high probability, based on the large cubic detector volume related to cylindrical detector volume. The vertical line in fig. 6 shows the position of the intersection and it clears the line more to the low energy region as the values of the lateral distance ρ increased.

The detection efficiency process is considered to be an important issue in the field of gamma-ray spectroscopy, where the detection efficiency indicates the probability of detection for the gamma-ray photons. A correct and exact calibration process is extremely significant for the quantitative measurement of an unknown sample. The perfect radiation detection setup must have a high quality of energy resolution with high efficiency as well, which is almost not easy to have in realistic applications, a negotiation is usually necessary [30]. The detector size and the gamma-ray photon interaction represent the main factors that con-

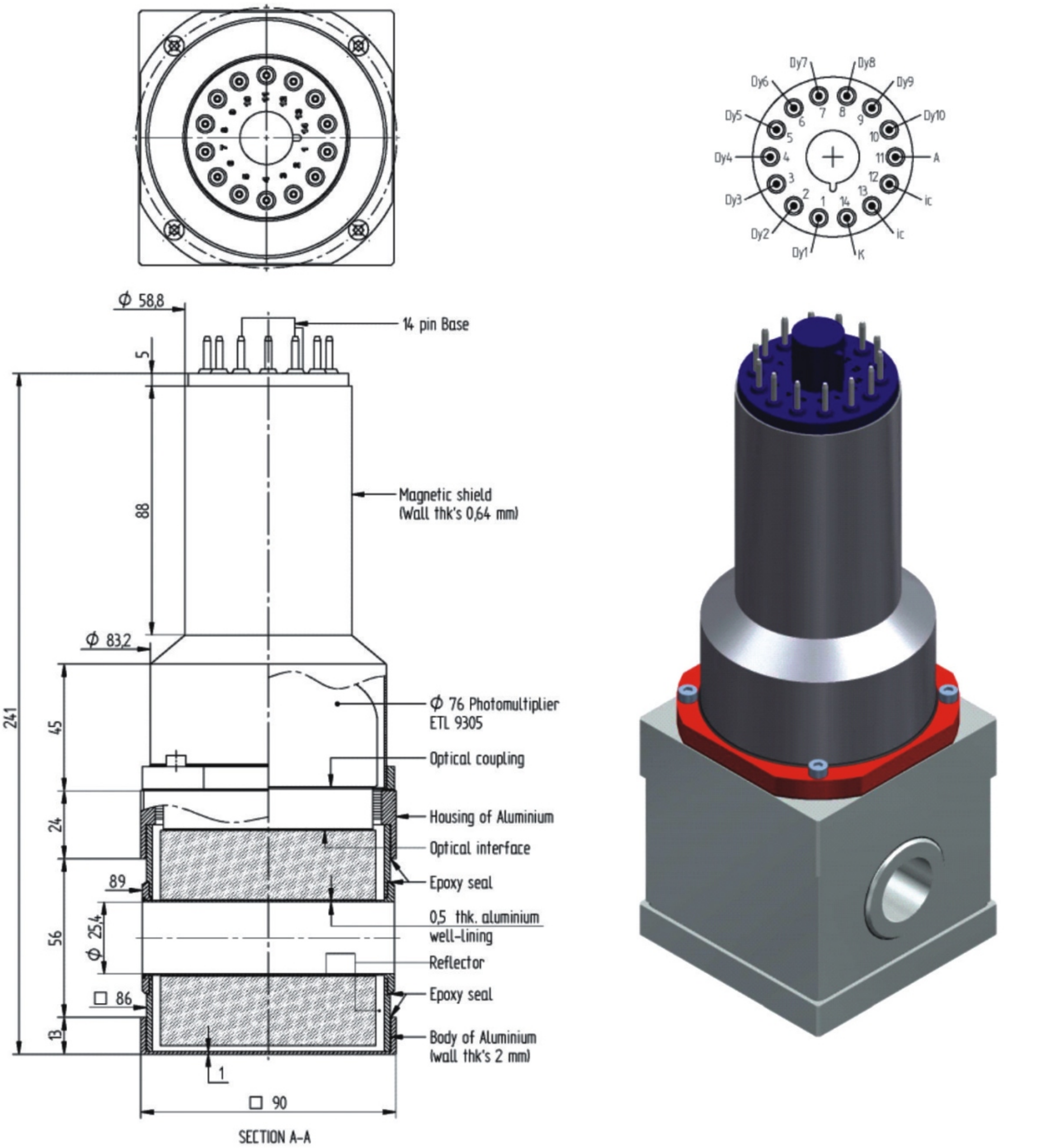

Figure 4. Geometrical drawing for cubic NaI(Tl) scintillation crystal with a side cylindrical hole enclosing connections of PM tube [31]

trol the detection efficiency. A limitation of the efficiency is that it is modified each time as the detection geometry setup is varied. To construct the efficiency values approximately independent of the detection geometry, the intrinsic efficiency was defined as the number of pulses detected over the number of radiation quanta incidents on the detector. Currently, the detector efficiency can be explained per number of gamma-ray photons incident on the detector material, therefore the power of the detection geometry is a good deal comfortable and the efficiency is almost self-governed by the geometry. However, at a very close source-to-detector setup distance, the intrinsic efficiency may vary significantly due to changes in the path length through the detector and therefore, care should be taken in this situation. If all the signals from the detector were considered depending on the interactions concerned with every gamma-ray photon in the whole spectrum, in this situation the total efficiency will be applied. The total efficiency $\varepsilon_{T(\text{Point Hole})}$ of the cubic scintillation crystal with a side cylindrical hole in case of non-axial point sources located inside the detector hole for each lateral distance ρ was tabulated in tab. 2.

Those values were calculated at energies starting from 0.05 MeV up to 3.00 MeV. In general, the total efficiency $\varepsilon_{T(\text{Point Hole})}$ shows decreasing in its values as the lateral distance ρ if its gamma-ray photon energies

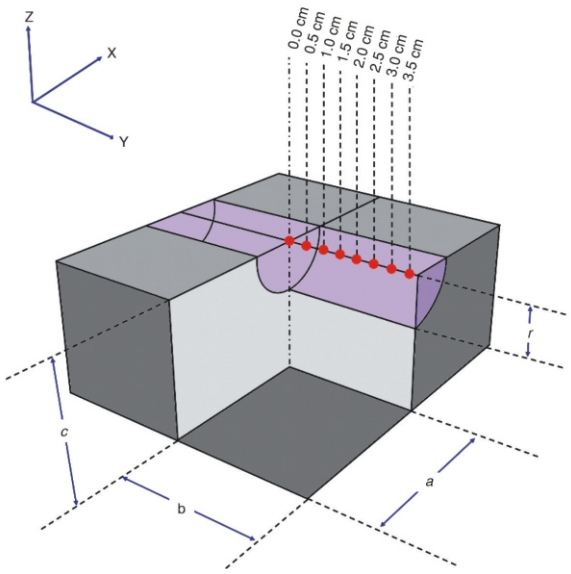

Figure 5. Radioactive sources were placed at eight positions on the detector hole major Y-axis

Table 1. Variation of geometrical solid angle $_{G(Hole)}$ and the geometrical efficiency $\varepsilon_{G(Hole)}$ as a function of lateral distance ρ

Lateral distance ρ [cm]	Geometrical solid angle $_{G(Hole)}$		Geometrical efficiency $\varepsilon_{G(Hole)}$	
	Cubic detector	Cylindrical detector [30]	Cubic detector	Cylindrical detector [30]
0.0	11.915	11.694	0.948	0.931
0.5	11.811	11.546	0.940	0.919
1.0	11.678	11.458	0.929	0.912
1.5	11.525	11.221	0.917	0.893
2.0	11.094	10.767	0.883	0.857
2.5	10.469	10.044	0.833	0.799
3.0	9.381	8.953	0.747	0.712
3.5	7.840	7.185	0.624	0.572

increased. In addition, there are comparative studies with the values of the cylindrical detector [30] as well in tab. 2 at the same positions and energies. At all the values of the lateral distance ρ the value of total efficiency $\varepsilon_{T(\text{Point Hole})}$ for the cylindrical detector till the gray color cells in tab. 2 is the higher one compared to the cubic detector values, while after the gray color cells the situation was reversed. This may be due to the difference in detector volumes, which leads to a change in the intrinsic efficiency of each detector from the others. The intrinsic and total efficiencies can be based on each other with the possibility of the incidence of gamma-rayphotons on the detector material. The geometrical solid angle can be used and is used to indicate this possibility.

Based on the ACT, fig. 7 represents a special comparison between the average path length $\overline{d}_{(\text{Point Hole})}$ through the cubic and the cylindrical detectors [30]. It was found that as the ρ/b ratio, along with the hole detector's Y-axis, increases on both sides from the hole center, the average path length $\overline{d}_{(\text{Point Hole})}$ for a cylindrical detector decreases, while its values slightly remains constant in the case of the cubic detector. This can be due to the symmetric shape around the hole in both detectors and the difference in both shapes, as well.

Figure 8 represents the variation of the geometrical efficiency $\varepsilon_{G(Hole)}$ concerning the ρ/b ratio along the hole detector's Y-axis. It is found that the geometrical solid angle $\varepsilon_{G(Hole)}$ for both cubic and cylindrical detectors [30], has its maximum value at the center of the hole, while it was decreased to reach its minimum values at both ends of the hole, within both detectors. Moreover, the values for the cubic detector are much higher than for the cylindrical detector, due to an insignificant difference in the dimensions.

CONCLUSION

The ACT for the calculation of the pure geometrical solid angle, the geometrical efficiency, and the total efficiency of cubic scintillation crystal with a side cylindrical hole, is introduced in this work. The method is depending on the diagram of the original possibility of gamma-ray photon detection, which explains the physical form for the detection in the real, particular studied conditions. In addition, it reproduces the applicable characteristics of the detection procedure. This possibility is based explicitly on the path of the gamma-ray radiation, allowing for the improvement and wider application of models than the traditional ones, depending on the integration process. This method is considered to be a simple and extremely fast process for calibration procedures. This style of detector is perfect for fuel rods and pipeline examining reasons, besides the radio-immunoassay performance. The path lengths of gamma-ray photons were studied across the detector and discussed over all the calculations, besides the photon average path length inside the detector medium itself. The technique is suitable for any type of source-to-detector design, where it can be useful, after several adjustments, to determine the full-energy peak in the case of a cylindrical radioactive source located inside or outside the detector hole. This will be exposed in a future paper jointly with a new measuring evidence. The outcomes show good quality of its behavior and a helpful approach for the novel detectors production, which can be used for measuring radiation at low-level.

AUTHORS' CONTRIBUTIONS

M. S. Badawi and A. A. Thabet carried out the theoretical formulas and the numerical validation testing. Both authors wrote the article together. In addition, all the authors arranged, reviewed, and discussed the results within the article.

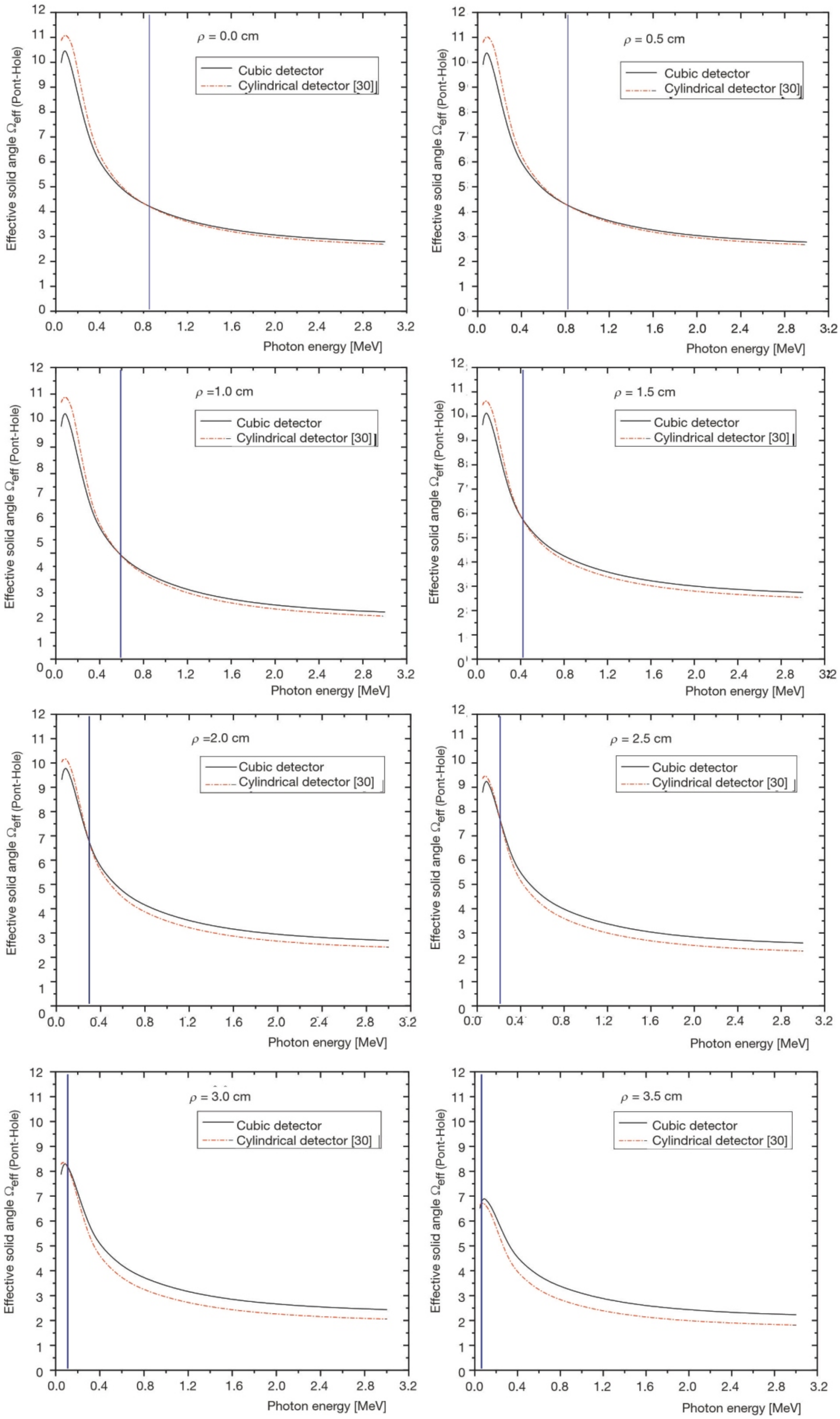

Figure 6. Comparison between the effective solid angle $_{\mathrm{Eff(Point\text{-}Hole)}}$ for the cubic and the cylindrical detector as a function of photon energy and the lateral distance ρ

Table 2. Comparison between the total efficiency $\varepsilon_{T(\text{Point} \quad \text{Hole})}$ for the cubic and the cylindrical detector as a function of photon energy and the lateral distance ρ

Lateral distance ρ [cm]	Total efficiency, $\varepsilon_{T(\text{Point} \quad \text{Hole})}$									
	Axial ρ = 0.0		Non-Axial ρ 0							
			0.5		1.0		1.5			
	Cubic detector	Cylindrical detector [30]	Cubic detector	Cylindrical detector [30]	Cubic detector	Cylindrical detector [30]	Cubic detector	Cylindrical detector [30]		
Photon energy [MeV]										
0.05	0.795	0.865	0.789	0.859	0.779	0.851	0.768	0.833		
0.06	0.821	0.875	0.814	0.870	0.805	0.860	0.794	0.842		
0.08	0.835	0.882	0.829	0.877	0.819	0.867	0.808	0.846		
0.10	0.828	0.882	0.821	0.878	0.813	0.866	0.803	0.844		
0.15	0.771	0.855	0.765	0.850	0.760	0.835	0.750	0.807		
0.20	0.692	0.769	0.687	0.764	0.684	0.748	0.675	0.552		
0.30	0.553	0.590	0.549	0.586	0.548	0.573	0.540	0.552		
0.40	0.470	0.488	0.466	0.485	0.466	0.475	0.459	0.458		
0.60	0.386	0.391	0.384	0.398	0.383	0.381	0.379	0.368		
0.80	0.342	0.342	0.340	0.340	0.340	0.333	0.336	0.322		
1.00	0.312	0.309	0.310	0.307	0.310	0.301	0.307	0.291		
1.20	0.290	0.285	0.288	0.283	0.288	0.278	0.285	0.269		
1.40	0.274	0.268	0.272	0.266	0.272	0.261	0.269	0.252		
1.60	0.261	0.254	0.259	0.253	0.259	0.248	0.256	0.240		
1.80	0.251	0.244	0.249	0.242	0.250	0.238	0.247	0.230		
2.00	0.243	0.236	0.242	0.235	0.242	0.230	0.247	0.230		
2.20	0.237	0.230	0.236	0.228	0.236	0.224	0.233	0.217		
2.40	0.232	0.224	0.231	0.223	0.231	0.219	0.228	0.212		
2.60	0.228	0.220	0.227	0.219	0.227	0.215	0.224	0.208		
2.80	0.225	0.217	0.224	0.216	0.224	0.212	0.221	0.205		
3.00	0.222	0.214	0.221	0.213	0.221	0.209	0.219	0.202		

Lateral distance ρ [cm]	Total efficiency $\varepsilon_{T(\text{Point} \quad \text{Hole})}$							
	Non-Axial ρ 0							
	2.0		2.5		3.0		3.5	
	Cubic detector	Cylindrical detector [30]	Cubic detector	Cylindrical detector [30]	Cubic detector	Cylindrical detector [30]	Cubic detector	Cylindrical detector [30]
0.05	0.742	0.799	0.700	0.745	0.627	0.659	0.519	0.528
0.06	0.766	0.807	0.723	0.752	0.648	0.665	0.537	0.534
0.08	0.780	0.810	0.737	0.753	0.662	0.664	0.549	0.535
0.10	0.776	0.806	0.733	0.746	0.659	0.656	0.549	0.530
0.15	0.727	0.764	0.688	0.702	0.624	0.616	0.527	0.504
0.20	0.656	0.680	0.622	0.623	0.568	0.549	0.490	0.457
0.30	0.526	0.522	0.502	0.481	0.464	0.428	0.410	0.365
0.40	0.448	0.433	0.429	0.401	0.398	0.359	0.356	0.309
0.60	0.370	0.349	0.355	0.324	0.331	0.292	0.299	0.254
0.80	0.328	0.306	0.315	0.284	0.295	0.257	0.267	0.224
1.00	0.300	0.277	0.288	0.258	0.270	0.233	0.245	0.204
1.20	0.279	0.256	0.268	0.238	0.251	0.216	0.229	0.189
1.40	0.263	0.240	0.253	0.224	0.237	0.203	0.216	0.178
1.60	0.251	0.228	0.241	0.213	0.227	0.193	0.207	0.170
1.80	0.242	0.219	0.232	0.204	0.218	0.186	0.199	0.164
2.00	0.234	0.212	0.225	0.198	0.212	0.180	0.194	0.158
2.20	0.229	0.206	0.22.	0.193	0.207	0.175	0.189	0.154
2.40	0.224	0.202	0.215	0.188	0.202	0.171	0.185	0.151
2.60	0.220	0.198	0.212	0.185	0.199	0.168	0.182	0.148
2.80	0.217	0.195	0.209	0.182	0.196	0.166	0.180	0.146
3.00	0.214	0.192	0.206	0.180	0.194	0.164	0.178	0.144

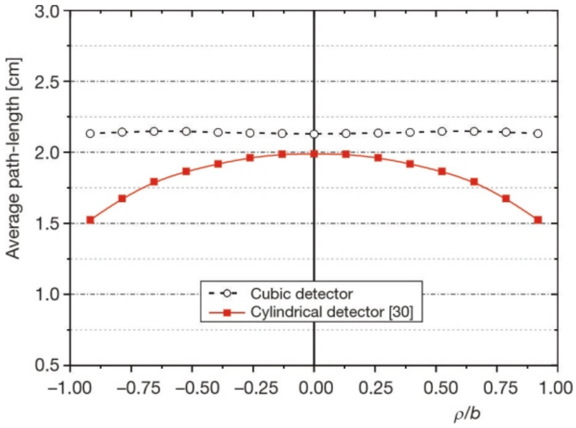

Figure 7. Comparison between the average path length $d_{(\text{Point} \quad \text{Hole})}$ through the cubic and cylindrical detectors

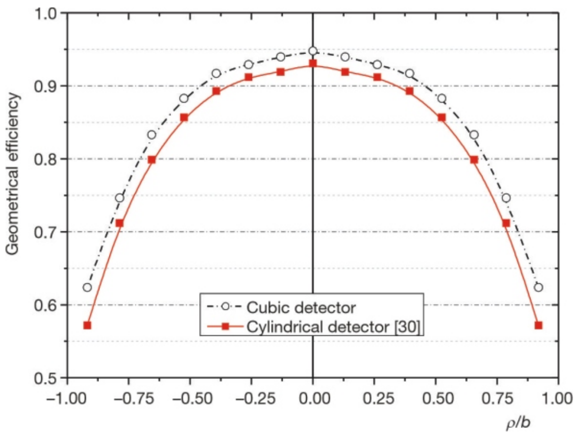

Figure 8. A variation of the geometrical efficiency $\varepsilon_{G(\text{Hole})}$ with the ρ/b ratio along the hole detector's Y-axis

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DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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АНАЛИТИЧКА ТЕХНИКА ПРОРАЧУНА ЕФЕКТИВНОГ ГЕОМЕТРИЈСКОГ ПРОСТОРНОГ УГЛА И ЕФИКАСНОСТИ КУБИЧНОГ СЦИНТИЛАЦИОНОГ КРИСТАЛА СА БОЧНОМ ЦИЛИНДРИЧНОМ РУПОМ

У области спектроскопије гама-зрачења, радиометријско испитивање малих количина природних узорака је изузетно значајно. Због тога се калибрација спектрометрије гама-зрачења мора припремити са добром прецизношћу за неколико енергија, матрица извора или узорака и облике простора од извора до детектора. У овом рукопису разматра се нова, некомпликована техника аналитичког прорачуна за одређивање ефективног геометријског просторног угла и ефикасност кубичног сцинтилационог кристала са бочном цилиндричном рупом. Прорачуни се могу обавити коришћењем једноставне методе, са неколико суштинских ограничења, која описује радиоактивне тачкасте изворе смештене унутар бочне цилиндричне рупе и високоефективни кубични NaI(Tl) детектор, заједно са ниским позадинским зрачењем. Техника се заснива на аналитичкој формули са просторним углом за систем детекције, користећи тачну једначину за шупљину детектора, удружену са грубим формулама које контролишу интеракције у извору гама-зрачења и материјалима унетим између извора и гама-спектрометрије зрачења. Ова нова техника није ограничена на одређене изворе, јер више облика извора може одговарати великом хомогеном броју тачкастих извора те геометрија детектора може бити представљена као скуп тачака на граници. Техника једноставно може бити корисна у будућности за постизање максималне ефикасности у пику енергије, што представља изазов за развој нискоенергетске гама-зрачне спектроскопије.

Кључне речи: аналитички прорачун, кубични сцинтилациони кристал, бочна цилиндрична рупа, геометријски просторни угао, ефикасност детектора