POWER LAW QUANTITIES BASED ON THE DISTRIBUTIONS OF TIME INTERVALS BETWEEN SUCCESSIVE NEUTRON COUNTS

by

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This study deals with the unfolded quantities based on the time intervals between successive neutron events from the aspect of the power law. Neutrons from spontaneous fission in special nuclear material induce fission in most cases. In this study, it was demonstrated that the inverse value of the number of induced neutrons decreases following a power function with the increasing radius of a plutonium metal sphere. In addition, it was considered an increase in the neutron background level with increasing altitudes. The inverse value of the mean neutron counting rate depending on altitudes can be described with a power function merely for higher elevations. A linear relationship was obtained by plotting the quantities on logarithmic axes against each other indicating generally a power law relationship for both investigated phenomena. The results of this study showed a connection between two seemingly unrelated neutron phenomena through power laws based on the distributions of time intervals between successive neutron counts. The empirical evidence implies that a connection between the observed quantities in a log-log plot is unchanged except for a multiplicative constant.

Key words: power law, neutron background, plutonium sphere, time interval

INTRODUCTION

The existence of power law was empirically proved for numerous natural and man-made phenomena but there is a lack of studies regarding power laws in nuclear science and engineering [1-3]. According to [4] every phenomenon can be represented mathematically and based on the typology of analogous phenomena every phenomenon can be reliably represented. The power law can imply a relationship between two quantities so that small changes in one quantity can cause a large change in the other, independently of their initial size. They can reveal regularity in the properties of highly complex systems so the changes between phenomena at different scales are independent of particular scales. It implies the self-similar property characteristic for power-law relationships so that seemingly unrelated systems can be connected if some data follow a power law relation. In the current study, we have analysed the altitude effect as the main source of spatial variation of the neutron background level as well as the mean time between induced neutrons depending on the Pu sphere mass and their connection to a power law.

Special nuclear materials (SNM) such as ²³⁹Pu, ²³⁵U, and ²³³U could be used as a basic component of nuclear explosives and therefore, their detection and identification are of high importance in the field of nuclear safeguards and homeland security. The SNM emits multiple neutrons and gamma rays correlated in time via spontaneous fission (SF) and neutron-induced fission (IF). By expectations, fission chain length increases with increasing mass of SNM [5, 6]. Several fast neutrons are promptly emitted in each fission event and due to their intensity and penetrability, such prompt neutrons are the most useful for passive assay of fissile materials. The fission event time is of interest in various applications [7, 8].

A bare Pu metal sphere consisting of 20 % of 240 Pu and 80 % of 239 Pu was used as a benchmark for the number (multiplicity) distribution of neutrons and gamma rays emitted from a fissile source [6, 9]. This composition of a Pu sphere was used more from a conceptual point of view. In the present study, we simplified the Pu sample description for clarity by considering only 239 Pu and 240 Pu mixtures in pure metallic form, for which the (α, n) contribution is negligible. In

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future work, we will take into account the presence of a small amount of other radionuclides in plutonium samples. The IF neutrons can be produced in such a sample due to neutrons with energies sampled from the Watt energy spectrum. We have performed the Monte Carlo (MC) calculations of the number of induced neutrons escaped from a bare plutonium metal sphere with varying mass by using the MCNPX-PoliMi code [10]. The number of neutrons emitted due to fission events as well as their energies are sampled from the multiplicity distribution and the Watt energy spectrum, respectively [11]. The dependence of the energy distribution on the number of neutrons emitted in the individual fission event was included in recent models [12].

The other phenomenon that we have investigated regarding power law is related to the neutron background level as a function of altitudes above sea level. We have measured the mean time between neutron counts depending on elevations in a range between 251 and 2067 m in Bosnia and Herzegovina. The preliminary results obtained have shown large fluctuations in the investigated altitude range. The results of mapping of neutron dose rate at 36 sites in Croatia in an altitude range from 2 to 988 m above sea level have shown similar behaviour [13]. The available results [14] on neutron counting rate depending on altitudes obtained with the mini neutron monitor (MNM) in Mexico in a wider range of higher altitudes above 2000 m up to about 4500 m as well as the available data measured with large neutron monitors (NM) at several global monitoring stations and at a few Antarctic stations [15] at heights above 2000 m were analysed from the aspect of power law behaviour.

There is a deficiency of studies on the neutron background measurements at altitudes up to about 2000 m. Following the ICRP 60 recommendations, there is a need for investigation of the neutron contribution to public dose [16]. Natural sources of ionizing radiation include two basic components such as cosmic and terrestrial radiation [17]. The main sources of gamma terrestrial radiation (TR) are the 40K, 238U, and ²³²Th long-lived radionuclides. The neutron contribution due to spontaneous nuclear fission of the three naturally occurring terrestrial radionuclides in the soil such as ²³⁸U, ²³⁵U, and ²³²Th can be neglected considering their low rates and natural abundances [18]. The total neutron flux variations occur in space and time, depending on the atmosphere conditions, the Earth's crust itself, and different geodynamical processes in the Earth's crust [19].

Cosmic radiation from the Earth's atmosphere [20] can be widely divided into primary and secondary components. The primary cosmic radiation consisting mostly of protons (87%) includes other charged particles of high energies (11% α particles, about 1% nuclei between beryllium and iron, and about 1% electrons). Secondary cosmic radiation (SCR) is produced

when primary particles interact with atmospheric gas nuclei [21]. The cascading effect, happening in the uppermost 10 % of the atmosphere, continues until the particles lose the energy required to create further secondary particles. The neutron component of cosmic radiation depends strongly on altitude [22] via attenuation of the neutrons as they propagate through the atmosphere towards Earth. Longer trajectories of neutrons through the atmosphere cause lower background radiation levels at lower altitudes [23]. Due to the strong penetrability of muons, photons, and neutrons through air, they can reach the ground level directly. The level of variation of the neutron background depends on a specific location and local environmental conditions.

To achieve a deeper understanding of the observed phenomena we have investigated the possibility of establishing a connection between them by using a mathematical relationship between two quantities, where a relative change in one quantity results in a proportional relative change in the other quantity, independent of the initial size of those quantities. The main goal of this study was to investigate interrelation through power law between two different neutron phenomena such as the altitude effect on variation of the neutron background level and the mean time between induced neutrons depending on the plutonium sphere mass, *i. e.*, between two seemingly unrelated phenomena.

MATERIALS AND METHODS

The MCNPX-PoliMi code [10] based on the MC method was used for the numerical simulation of all possible neutron-nucleus interactions within a plutonium metal sphere, in particular of the simulation of spontaneous and neutron-induced nuclear fission. The MC code used in the numerical simulations includes new models of neutron and gamma-ray emissions from spontaneous and induced fission for several isotopes of interest such as ²⁴⁰Pu and ²³⁹Pu. The most defining characteristic of SNM, that are of interest in nuclear safeguards and non-proliferation applications, is neutron-induced fission. The MC model includes the number, spectral, directional, and temporal distributions of neutrons and gamma rays of the various radionuclides as well as the full description of the multiplicity distributions of neutrons or gamma rays from nuclear fission.

Measurements of the neutron dose rates in Croatia [13] at 36 sites from the sea level up to about 1000 m were performed with the CR-39 track etch detector calibrated by the CERN-EUhigh-energy Reference Field (CERF) facility. The data on altitudes and neutron dose rates (given in tab. 1 of ref. [13]) were used in the current study for their further analysis regarding power law.

A network of ground-based neutron monitors (NM) has been used for monitoring the near-Earth cosmic ray

flux since the middle of the last century. To inter-calibrate the global network of NM, it was initiated in 2001 the development of a smaller and cost-effective version of the traditional NM, so-called the mini-NM [24, 25], that can be easily shipped and installed. The mini-NM's design has been continually optimized since then. The available data on the neutron counting rates in Mexico (mostly in the state of Veracruz) depending on elevation above sea level were obtained with the mini neutron monitor [14]. The dimensions of the mini-NM with all components are approximately 900 m (height) \times 700 mm (width) \times 800 mm (length). The gas pressure in a mini-NM detector is larger than in a standard NM64 detector partly to compensate for the reduced efficiency of the mini-NM compared to the standard NM64 of larger size.

The available data [15] obtained at a few existing monitors of the global monitor network with the International Geophysical Year (IGY) and Neutron Monitor (ND) were analyzed from the aspect of power law forms. The first neutron monitor was designed in 1951 and optimized with a new version consisting of 12 ¹⁰BF₃ counters for use during the International Geophysical Year (IGY) by 1957/8 [26]. The NM64 ¹⁰BF₃ super neutron monitor was designed in 1964 [27]. Since then, neutron monitor design has been constantly improved.

The current study deals with identifying the power law behaviour of the observed data based on the mean time between neutron events. When the probability of measuring a particular value of the quantity varies inversely as a power of that value, it can be said that the quantity follows a power law [2]. The existence of power-law forms has been proved in a wide variety of natural and manmade phenomena, from physics to finance. A power law can be described by the following eq.

$$p(x) = Cx^{-\alpha} \tag{1}$$

where C is a normalization constant, $\alpha > 0$ is the exponent of the power law, and p(x)dx is a probability of a taking value in the interval between x and x + dx. The α parameter gives the rate of decrease of probability when x increases. After taking logarithms of both sides, a straight line is obtained in a log-log plot.

RESULTS AND DISCUSSION

Mean time between neutron events depending on plutonium sphere mass

We have investigated a connection between the mean-time intervals between successive neutron events and plutonium spheres of different sizes. Events of nuclear fission, particle transport, multiplication, and absorption in the bare plutonium metal spheres with radii from 0.3 cm up to 1.7 cm with the step of 0.1 cm (systems are far away from criticality),

composed of 20 % of ²⁴⁰Pu and 80 % of ²³⁹Pu, and mass density of 15.9 gcm⁻³ were simulated by the MCNP-PoliMi code [10]. Since neutrons from (α, n) reaction are negligible for the metal plutonium sphere, neutrons from spontaneous fission induce fission in fissile materials [5]. Fissile materials sustain fission chain reactions in dependence on the composition, mass, and multiplication of the fissile material [7]. The average SF neutron multiplicity in ²⁴⁰Pu is about 2.16 [28] whereas the average IF neutron multiplicity in ²³⁹Pu is about 3.16 induced by neutrons with an energy of 2 MeV [29]. A location of a ²⁴⁰Pu nucleus undergoing the SF reaction is sampled uniformly within a plutonium sphere in the MC simulation. It is assumed uniformly distributed the direction of fission neutrons in space. The simulated data for the number of induced neutrons as a function of a plutonium sphere mass is shown in fig. 1(a) whereas fig. 1(b) shows the inverse value of this quantity relating to the mean-time intervals between neutron events. It can be seen that there is a straight line in a log-log plot, fig. 2, for these quantities with a slope of -0.47 with 95 % confidence bounds of (-0.48, -0.46) and adjusted $R^2 = 0.99$ indicating that almost all the variability was included in the linear regression model.

It is obvious from fig. 2 that a connection between the quantities in a log-log plot is unchanged except for a multiplicative constant. A power law distribution is the only distribution that is the same independently of the particular scale at which we observe the data. The power law is often called a scaling law since it describes the functional relationship between two physical quantities scaling with each other over a considerable interval. If different scales are multiplied by a common factor, scaling laws do not change implying universality [30].

Mean time between neutron counts as a function of altitudes

Mapping of the neutron dose rate in Croatia depending on elevation above sea level

Mapping of the neutron dose rate in Croatia at different altitudes from sea level up to approximately 1000 m was performed with the CR-39 track etch detector calibrated by the CERN-EUhigh-energy Reference Field (CERF) facility [13]. It demonstrated a significant positive correlation coefficient between the neutron dose rates and altitudes at 36 measurement sites. A log-log plot of the altitudes and the inverse values of neutron dose rates given in tab. 1 of ref. [13] are presented in fig. 3. It can be noticed large fluctuations of the experimental data around the straight line for altitudes up to 1000 m above sea level. The slope of the line of -0.06 with 95 % confidence bounds of (-0.09, -0.02)

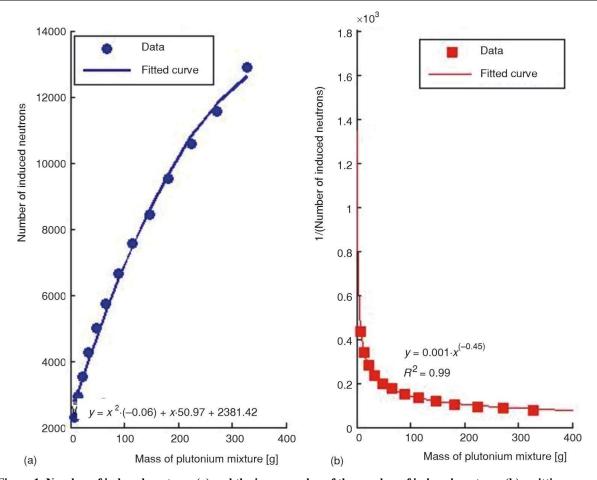


Figure 1. Number of induced neutrons (a) and the inverse value of the number of induced neutrons (b) emitting from a bare plutonium sphere of various masses

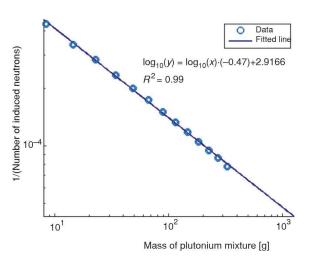


Figure 2. Mean time intervals between induced neutrons as a function of plutonium sphere mass in a log-log scale

indicates a weak dependence of neutron dose rates on altitudes, in particular at an altitude of about 200 m where most measurements were carried out. However, it can be seen that there is a decreasing trend of the mean time between detected counts with increasing altitudes which is in line with the expectations.

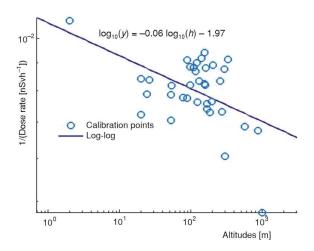


Figure 3. Mean time between neutron counts depending on altitudes at 36 measurement sites in Croatia [13]

Altitude survey by the MNM in Mexico

We have analysed the data (altitudes and the mean counting rate corrected by atmospheric pressure) obtained with the MNM during the altitude survey, which are given in tab. 1 of ref. [14]. The altitude

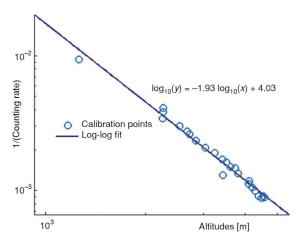


Figure 4. The mean time between neutron counts depending on altitudes measured with the MNM mostly in the state of Veracruz (Mexico) between about 1000 and 4600 m above sea level

survey was performed mainly in the state of Veracruz at an altitude range from the sea level up to about 4600 m and the survey latitude range was less than one degree. Figure 4 shows the inverse values of the mean neutron counting per minute depending on altitudes above 1000 m to 4600 m in a log-log scale. The straight line is obtained by using the least squares fitting model. The evaluated value of the exponent alpha is -1.93 with 95 % confidence bounds of (-2.04)-1.82) indicating that the mean time between neutron counts decreases with increasing altitudes in the given altitude range. However, it is known that just a few distributions from the real world follow a power law relation in their entire range, especially for smaller values of the variable. Figure 5 shows the mean time as a function of altitudes but in a narrower range, from 1000 up to about 4600 m. It can be seen that the measured data do not deviate significantly from a straight line in a log-log plot.

Neutron counting rates at global stations measured with the NM

The data available on the neutron counting rate and altitudes at global neutron monitoring stations were divided into a world map of existing monitors and the Antarctic bases above 2000 m.

We have analyzed the available data obtained at a few existing monitors (Haleakala, Climax, Mexico, Huancayo, Jungfraujoch 1, Lomnicky Stit, Alma Ata, Mt-Norikura) of the global monitoring network measured with the IGY and NM containing gas-filled detectors with length up about 2 m and the available data measured at a few monitoring stations in Antarctic (Amundsen-Scott, Concordia Dome C, D85 ski way, Dome Fuji, Kohnen, Mid-Point, Vostok) above 2000 m above sea level [15]. Figure 5 shows the inverse values of the neutron counting rate as a function of altitudes. It can be noticed that the mean time between

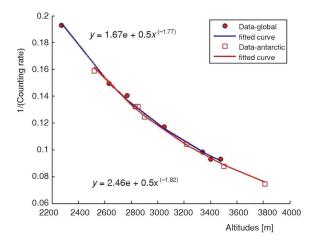


Figure 5. The inverse values of the counting rate for a few global and Antarctic monitoring bases measured with the NM depending on elevations above sea level

neutron counts decreases with increasing altitudes for both groups of the neutron monitoring data. Figure 6 shows the same quantities in a log-log plot. It can be seen approximately the same slope of the straight lines (for global monitoring bases it is about -1.77 and for Antarctic monitoring bases is about -1.86).

The causal relationships between the variables related to the power law, considered in this paper, could be the subject of further investigation.

CONCLUSIONS

The mean time between successive neutron events due to two different neutron phenomena was examined in the context of power laws. The results of the MC calculations of the complex processes in a bare Pu metal sphere with a mixture of 20 % of ²⁴⁰Pu and 80 % of ²³⁹Pu have shown that the inverse value of the number of induced neutrons relating to the mean time between neutron events decreases with increasing radius of Pu metal sphere. We found that the inverse value of the number of induced neutrons and the mass of the Pu mixture were connected through power law almost perfectly. It implies some additional characteristics which will be considered in future work. An increase in the neutron background level and consequently a decrease in the mean-time interval between neutron counts with increasing altitudes was additionally analysed. The large fluctuations of the experimental data around the straight line were observed in the altitude range up to 1000 m above sea level whereas the measured data for altitudes above 2000 m do not deviate significantly from a straight line in a log-log plot. This finding is in agreement with the fact that just a few distributions from the real world follow a power law in their entire range, especially for smaller values of the variable.

The current study shows that two seemingly unrelated phenomena can be connected through power laws

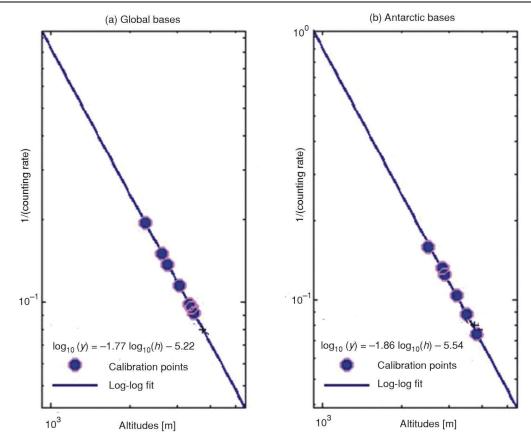


Figure 6. The inverse values of the counting rate for a few global (a) and Antarctic monitoring bases (b) measured with the NM in the dependence on altitudes

based on the mean-time intervals between successive neutron events. Power laws can reveal regularity in the properties of complex systems so the changes between phenomena at different scales are independent of particular scales. It was demonstrated that a connection between the quantities as the measurable physical property associated with the phenomenon, in a log-log plot is unchanged except for a multiplicative constant. It is planned in the next stage of research to investigate the mechanism driving the investigated quantities to follow a power law.

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AUTHORS' CONTRIBUTIONS

The majority of this work was initiated and performed by the lead author (S. Avdic) whereas the co-authors contributed with interesting discussions, analysis of the results obtained, and writing the manuscript.

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ВЕЛИЧИНЕ СТЕПЕНЕ ФУНКЦИЈЕ ЗАСНОВАНЕ НА РАСПОДЕЛИ ВРЕМЕНСКИХ ИНТЕРВАЛА ИЗМЕЂУ СУКЦЕСИВНИХ НЕУТРОНСКИХ ИМПУЛСА

Ова рад бави се проучавањем развијених величина на бази временских интервала између сукцесивних неутронских догађаја са аспекта степене функције. У већини случајева, неутрони емитовани при спонтаној фисији специјалних нуклеарних материјала индукују фисије. У овој студији је демонстрирано да инверзна вредност броја индукованих неутрона опада са повећањем радијуса плутонијум сфере следећи степену функцију. Додатно је размотрен пораст нивоа неутронског фонског зрачења са повећањем надморске висине. Инверзна вредност средњег броја неутронских импулса у јединици времена може се описати помоћу степене функције једино при вишим елевацијама. Добијена је линеарна зависност између величина на логаритамској скали индукујући закон степене функције за оба истражена феномена. Резултати ове студије показали су везу између два наизглед неповезана неутронска феномена на основу расподела временских интервала између сукцесивних неутронских импулса преко степене функције. Емпиријска евиденција имплицира да је веза између истражених величина у log-log дијаграму непромењена изузев за мултипликативну константу.